Acta Technica Napocensis: Civil Engineering & Architecture Vol. 58, No. 4 (2015)
Journal homepage: http://constructii.utcluj.ro/ActaCivilEng
Special Issue– *International Workshop in Architecture and Urban Planning*
Sustainable Development and the Built Environment

Generative Surfaces. Texture on Surface Influenced by a Vector Field

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(*Published online 28 February 2016*)

Abstract

We investigate a new kind of approach for architectural design in computational design environment, that takes into account physical characteristics of the material generating the form in the formation process of the texture upon a given surface. Recent development of tools and techniques, initially developed in the field of physics, structural and mechanical engineering for analysis and evaluation of physical behavior, are gradually being introduced in the field of architectural design as means of generating form, based on behavioral characteristics of the material. The article proposes the generation of a texture on a surface following two families of curves on a surface: the principal stress lines of a vector field projected on the boundary surface, and the equipotential curves of a force field imposed on a surface. In the first part, the research investigates the relationship between vector fields and the generated types of curves, meant for the comprehension and utilisation of the existing computational tools (such as Flowl for Grasshopper/Rhino software) for architectural form search. The second part of the research is a experimetal exploration of form based on the above notions of vector fields, and derived families of curves. The research shows the possible use of families of curves specific for a surface in architectural approach for experimental design as well for rationalization for fabrication purposes.

Rezumat

Investigăm o nouă metodă de abordare în proiectarea de arhitectură în cadrul mediului de design computațional, care ia în calcul caracteristicile fizice ale materialului în procesul de formare al unei texturi pe o suprafață. Noi unelte și tehnici, dezvoltate inițial în domeniul fizicii, ingineriei mecanice și de structuri pentru analiza și evaluarea unor fenomene și comportamente fizice sunt introduse treptat în domeniul proiectării de arhitectură, susținută de designul computațional, ca mijloace de generare a formei, bazată pe caracteristici și comportamente ale materialului. Articolul propune generarea unei texturi pe o suprafață curbată pe bazaa două familii de curbe: liniile principale de tensiune ale unui câmp vectorial proiectat pe suprafață și curbele de echipotență ale câmp de forță impus pe o suprafață. Liniile de tensiune principale sunt generate *pornind de la un câmp vectorial, plasat pe suprafață, cu origine și direcție. În prima parte a cercetării, urmărim clarificarea unor noțiuni legate de vectori și câmp vectorial , și legătura pe care aceștia o au cu anumite tipuri de curbe specifice pentru o suprafață. Aceste unelte se regăsesc în mediul de design computațional (de ex. Add-*onul *Flowl pentru software-ul Grasshopper/Rhino)ca mijloace de explorare formală. A doua parte a cercetării este o explorare experimetală asupra formei , bazată pe noțiunile de câmp vectorial și familii de curbe derivate. Cercetarea arată o posibilă utilizare a unor familii de curbe specifice pentru o suprafață dintr-o*

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perspectivă a designului experimental dar și pentru raționalizarea în scopul fabricării.

Keywords: Vector field, Stream lines, equipotential lines, tensor, form exploration.

1. Introduction

In the realm of computational design, new tools for exploring new families of form become available for the use of architects. Such tools, based upon notions of mathematics and physics are often not familiar for architectural understanding and use.

The research presented in this article, focused on the use of computational tools that imply the creation of fields of vectors and the associated, derived curves, for architectural use, has three objectives: the first aim is to familiarize the architect with the notions of vector fields and the associated families of curves, the second objective is to investigate possible use of the respective curves in order to create architectural form, and the third objective is to relate curves extracted from a vector field on a given NURBS surface, both as decorative approach, and as a more efficient structural approach.

2. Scalars, vectors, tensors

In general, vectors are used to model physical properties, such as the speed and direction of a fluid throughout space, or the strength and direction of a force, such as magnetic or gravitational force, that changes from point to point, and during time.

In vector calculus, a vector field is an assignment of a vector to each point in a subset of space. A vector field in plane can be visualized as a collection of arrows with a given magnitude and direction each attached to a point in plane. A vector field, when related to surfaces, associate an arrow to the surface at each point. The vectors can be tangent, normal to the surface, or they can have different angles with the surface.

The initial use of vector fields use is in physics, and solid mechanics. There are more kinds of ”fields”, used for representing different physical properties. *Scalar fields* are used to represent properties such as temperature, pessure and density. These properties are represented through *scalars*. *Scalars* are mathematical enteties that have magnitude, but no direction. *Vector fields* are used to represent force fields, such as gravitational force or electrostatic force field, or fluid velocity. *Vectors* are mathematical entities that have both magnitude and direction. In the euclidean Cartesian coordinate system, a *vector* is represented as: $\mathbf{V}=\alpha\mathbf{i}+\beta\mathbf{j}+\gamma\mathbf{k}$, where α,β,γ are the scalar (quantitative) component along the x,y,z, coordinate axes , and $\mathbf{i},\mathbf{j},\mathbf{k}$ are unit vectors along the same axes. A vector field is a position dependent vector, $\mathbf{v} = \mathbf{v}(r)$.

Tensors are mathematical entities that have a scalar component and two directions. They are represented through matrices of 2,3 to n dimensions. Common relations between vectors (dot product and cross product) give outputs such as the scalar product, or the direction perpendicular to the initial plane of the two initial vectors. In order to change the magnitude *and* directions *of* a vector, another mathematical entity is introduced, *the tensor* (**[1]** Kolecki**)**. Tensors are used to represent elastic stresses and elastic strains, (and momentum flux density) in a material object(**[2]**Delmarcelle). They take a vector as input and give as output another vector with different magnitude and orientation. Tensors generate an expression between physical input data and output data, such as the two quantities should be equal when measured in different coordinate systems.

The physical quantity that is measured by tensors is independent of a certain coordinate system. This means, that tensors have the property that they are invariant under coordinate transformation.

Tensors have different ranks or degrees, depending on the how many dimensions has an array. A scalar is a rank 0 tensor, that has only magnitude, but no direction. A vector is a tensor of rank 1, and in a cartesian system it has three coordinates. A tensor of rank two, or a dyad, in Cartesian system has 3^2=nine components. Note that not every scalar is a tensor, nor every vector is a tensor, as they must preserve physical quantities when measured in different coordinate systems.

3. The relation between vector fields and curves on surface. Representing tensor fields as curves on surfaces

One way of representing tensor data is by drawing lines of individual components, that look like level curves on a map. These images don't reflect the complexity of the structure of the tensor field (**[3]**Delmarcelle). In the case of 2d data, three plots are necessary to represent the tensor field, (one image for each component), and for the case of 3-D data, six or nine images are necessary to represent the tensor field. Additionally, when changing the orientation of coordinate axes, the representation also changes accordingly(**[4]**Delmarcelle).

One problem is how to represent a tensor field in a way that is invariant under rotation of coordinate axes. One possibility is to represent the tensor field as two *eigenvector* fields $\boldsymbol{v}_i$ with components $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$ perpendicular to each other. This is relevant for drawing principal stress lines on surfaces. When a double curves surface, a shell, changes shape, the local coordinate axes of the curve also change. Eigenvectors have the property that they don't change direction when the coordinate axes rotate or stretch (they remain invariant). The representation with eigenvectors allows a continuous representation of vectors placed at points distributed over the surface, from which a coherent texture of lines can be extracted. These lines are known in mechanics as ”trajectories of the principal stresses” or ”hyperstreamline trajectories”.

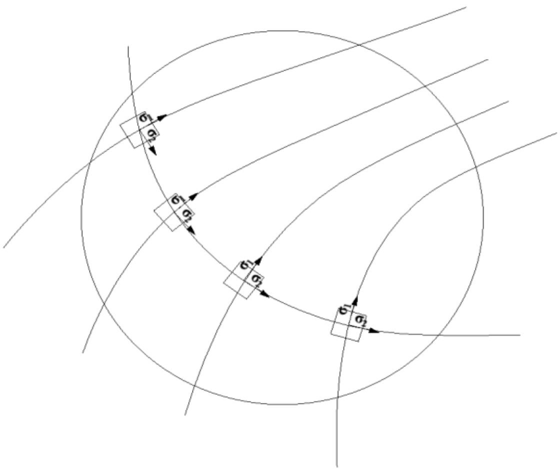

Figure 1. Vectors $\boldsymbol{\sigma_1}$ and $\boldsymbol{\sigma_2}$ at four points and the associated streamlines

We note $\boldsymbol{v}_1$ and $\boldsymbol{v_2}$ as and $\boldsymbol{\sigma_1}$ and $\boldsymbol{\sigma}_2$. In a plane, at each point across the surface, there act two principal stress components $\boldsymbol{\sigma_1}$ and $\boldsymbol{\sigma_2}$, oriented in the two principal directions of stress distribution. Following the variation of the principal directions from one point to another, one can trace a curve, a *principal stream line*. The stress components, $\boldsymbol{\sigma_1}$ and $\boldsymbol{\sigma_2}$ are tangent to the principal stream lines(**[5]**Antonescu**)**. There are two stream lines, S1 and S2 corresponding to each principal stress direction. Through every point of the surface there will be two such streamlines, perpendicular on each other and tangent to the surface, that can be considered trajectories of the main stresses (Fig.1).

4. Experimental approach

In the context of computational design, where form finding methods are a central focus of architectural research, there have been introduced tools for operating with vector fields in order to generate new kinds of forms, such as Flowl add-on for Grasshopper/Rhino software, linked in a greater or smaller extent to their original use in physics.

In the context of generating textures on free-form surfaces, usually the goal is to generate efficient structural grids that can describe in an optimal way the given NURBS surface. Such structural patterns can be achieved by identifying on the surface *principal stream lines* (curbe izostatice).

Consider a vectorfield $\boldsymbol{v(x,t)}$, where $\boldsymbol{x}$ is the position in space and t is the time. Streamlines at a moment t_0 are integral curves satisfying $\frac{d\vec{x}}{ds}=\vec{v}\left(\vec{x},t_0\right)$, where t_0 is a constant and s is a parameter measuring distance along the path. So extracting streamline curves on the surface is done by the integration of the equation above, with special techniques, such as Euler explicit scheme or as fourth-order Runge Kutta techniques.

”Flowl” is an add-on for Grasshopper/Rhino modelling platform that enables the extraction of principal stream lines out of a vector field with such a mathematical technique. Given a 3d vector field, with origin, scale and direction, the add-on extracts the stream lines of that vector field. The stream-lines are tangent in every point to the respective vector field.

Based on Rhino/Grasshopper modelling and scripting software platform, we conducted a set of experiments envolving vector fields. We initially generated a plane force field, with several „charge points”, that is points where one applies loads (in our case 5 points of charge, a force intensity , which orients the directions of lines, a decay value , which is a damping value for the force. In our case we used only a so called “attraction force” that generate relative uniform results and that splits space in 5 relatively uniform regions. Then we applied the given texture on a given NURBS surface.

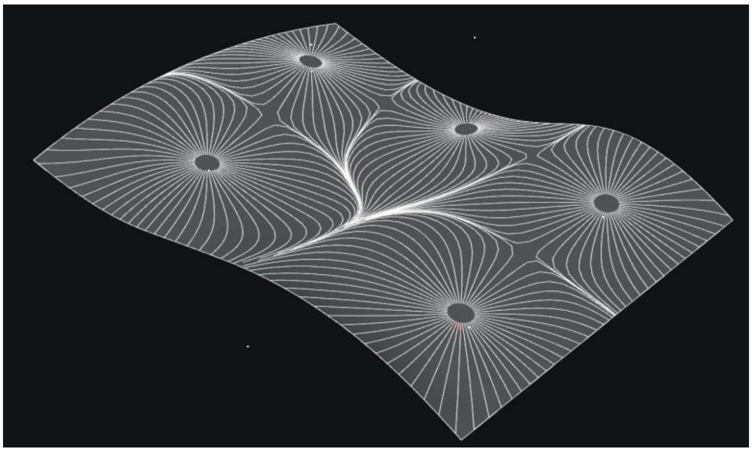

Figure 2. Force field applied on an arbitrary NURBS surface

The physical explanation of how the regions around charge points are formed can be found as follows: *„The tajectories of one vector field never cross each other, except at one point, called "degenerate point". Trajectories that do not meet at degenerate points tend to converge towards a few specific lines that emanate from degenerate points. We call such lines"separatrices".*(**[6]** Delmarcelle)*The separatrices separate the domain into regions where eigenvectors are equivalent to uniform fields. From this set of lines, one can infer the directions of vectors at any point in plane."*(**[7]** Delmarcelle)

We make a special note in order not to confuse "*charge points"* with *"degenerate points". Charge points* are in the center of the region, generating the force lines, while the degenerate points lye on the boundaries of the region, marking the division between regions, and they are not represented in Fig.2.

From Fig.2 one can realize that there is no explicit relationship between the overall geometry of the NURBS surface and the pattern that is meant to be part of a gridshell describing the surface. Therefore, we extended our investigation, searching for a more appropriate relation between the geometry of the surface and the geometry of the texture.

In the following example we start with a flat surface, which is transformed into a curved surface based on the logic of attraction points. The surface is subdivided into smaller patches, and the vertices (points) on the surface are moved in vertical direction proportional with the distance from the attraction points. The rule of movement is supplied by a combination of functions (parabolic and BSpline), in order to get a certain slope of the surface. This is a parametrical approach, which makes possible the generation of a family of surfaces based upon the variation of the parameters of the functions. This allows the generation of steeper or less curved surfaces, that is of possible shells with higher or lower profile and different, adaptable curvature, according to functional needs. Two such possible individuals are presented in Fig.3a,3b.

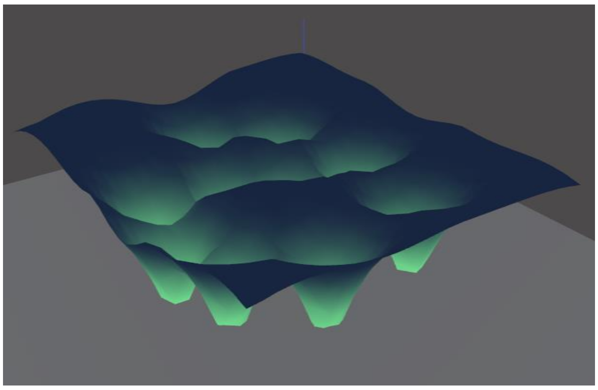

Fig. 3a

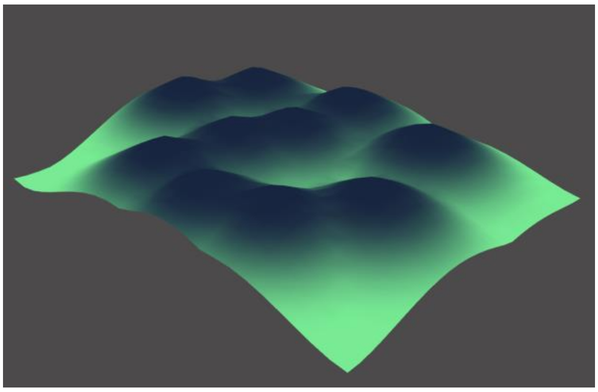

Fig. 3b

Figure 3. Possible individual out of parametrical description of surface

The next point aims to establish an explicit relationship between the geometry of the overall surface and that of the grid generated by the vector field. This is done by considering the attractor points of the surface as the point charge of the generated vector field. This means that points of local maxima or minima (with the highest or lowest levels) coincide with the point of application of force from the force field. In a built structure, the charge points may represent support points, or the points for placing the pillars, from which a canopy of radial ribs spread out in the direction of the force lines; this creates a canopy of cantilevered structures, where one pillar defines one region, and several regions create the overall structure (Fig.4a); this is the example where the charge points of the vector field coincide with points of local minima. The other way around, if the points of charge coincide with the local maxima, the generated structure resembles an addition of domes, where the points of charge coincide with the peak of the domes (Fig 4b). In this case as well, the slope of the geometry of the surface coincides with the regions of the streamlines derived from the vector field.

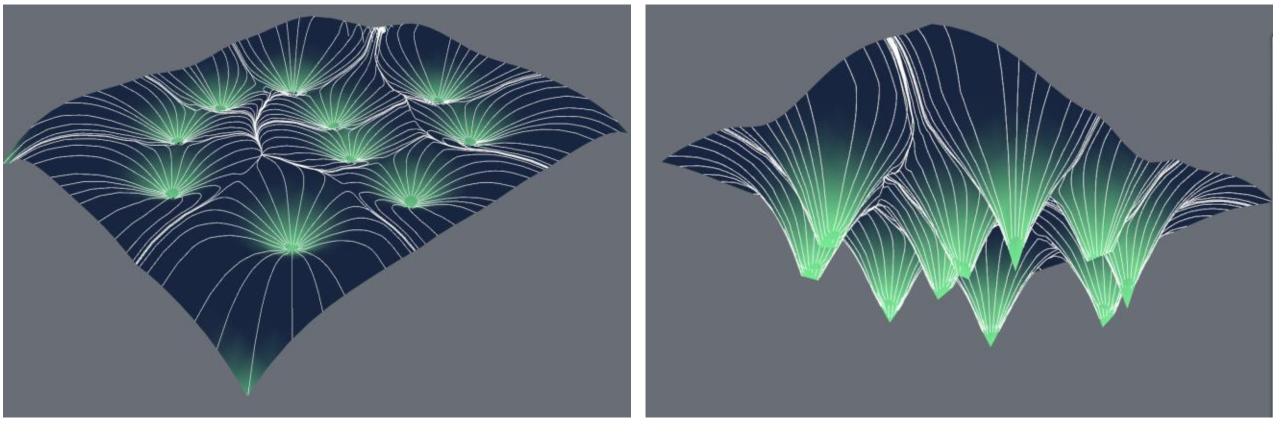

Figure 4a. Streamlines of vector field superimposed on surface geometry.
A cantilevered structure

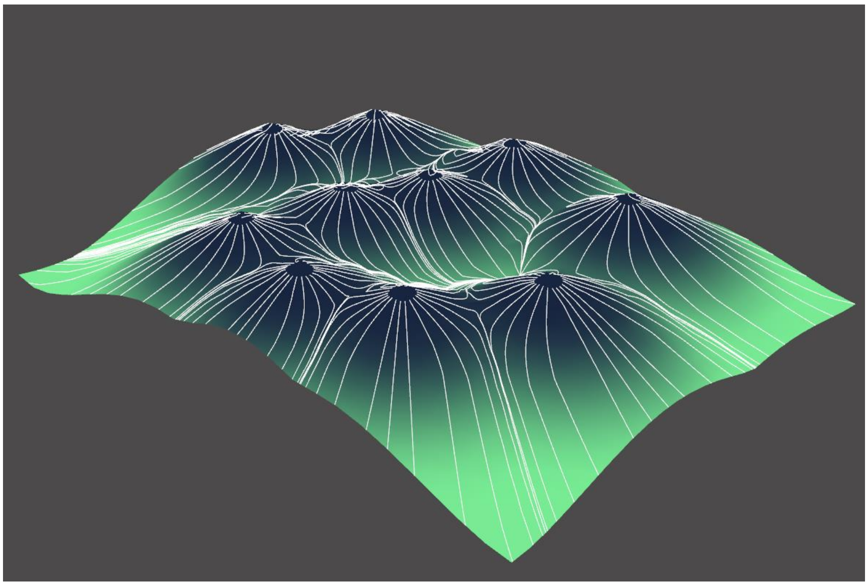

Figure 4b. Streamlines of vector field superimposed on surface geometry.
A structure of additional domes

The previous steps show a rationalization of a curved surface based on a texture imposed by the streamlines derived from a vector field. This generates a texture on the given surface which might be interpreted for fabrication purposes in two ways: if the shell is going to be made out of continuous material such as metal sheets, the texture is useful for extracting panel sheets, possibly in form of ruled surfaces, that is surfaces that have curvature in only one direction. If on the other hand, the shell is meant to be made as a gridshell, that is a frame structure that describes the surface and acts like a load carying structure, with additional glazing or simple panels, the curves derived from the vector field and applied to the surface may act as the radial ribs of the frame. In this case, we still need a second family of curves, which indicate the meridian ribs. Therefore, we introduce a second class of curves, the *equipotential* curves. *Equipotential* or *isopotential* in mathematics and physics refer to a region in space where every point in it is at the same potential (**[8]** Weisstein).

These notion is used mostly in electrostatics, where electrostatic equipotential curves are measured between two electrically charged spheres. Roughly described for the purpose of understanding the notion, the curves surround the charge source in approximate concentrical surfaces. The charge source is perpendicular to the equipotential surface of the sourse: "*the electric field is perpendicular to the equipotential surface of the electric potential*".

In our case, the equipotential lines are extracted from a vector field, resulting in concentrical curves surrounding the charge point. We illustrate the first results for comprehending the orientation of the equipotential lines related to the source of the field (Fig.5).

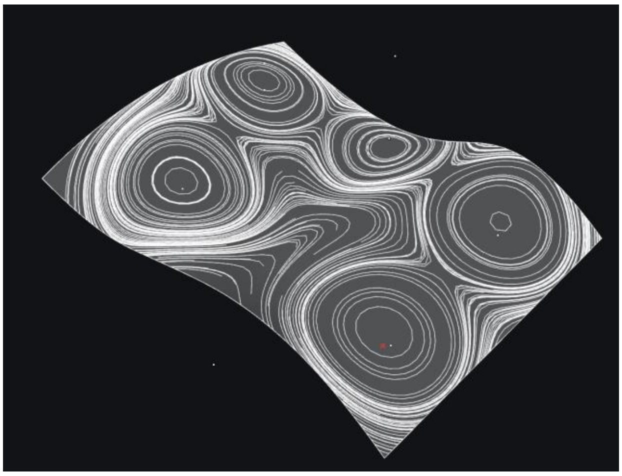

Figure 5. A generic NURBS surface with equipotential lines related to five point charges

This family of curves supplies a convenient approach in the case of the addition of multiple dome like structures, articulating the valley areas between the lumped areas over a continuous curved surface.

Superimposing the equipotential curve texture over the streamline texture generates a grid with a rational geometry that can be approximated by quadrangular panels or frames (Fig 6). Further on, if we superimpose the qudrangular curve texture over a surface geometry where the peaks (Fig.7), (or the valley points), correspond to the charge points of the force field we obtain a continuous, fluid surface, which can be parametrically rationalized and controlled for fabrication and construction purposes with the aid of vector field tools.

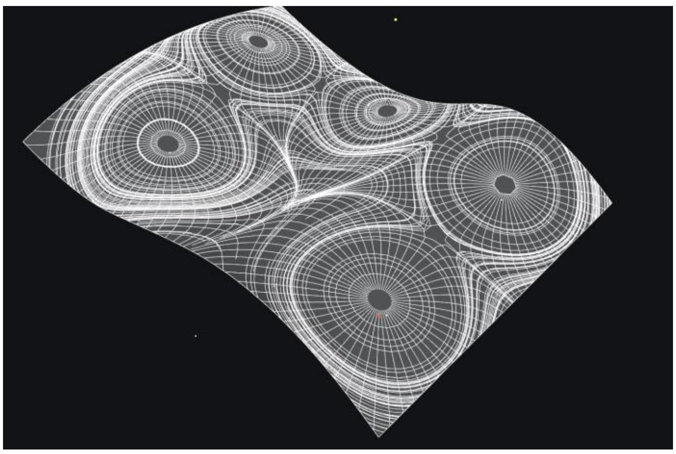

Figure 6. A generic NURBS surface with equipotential and stream lines texture

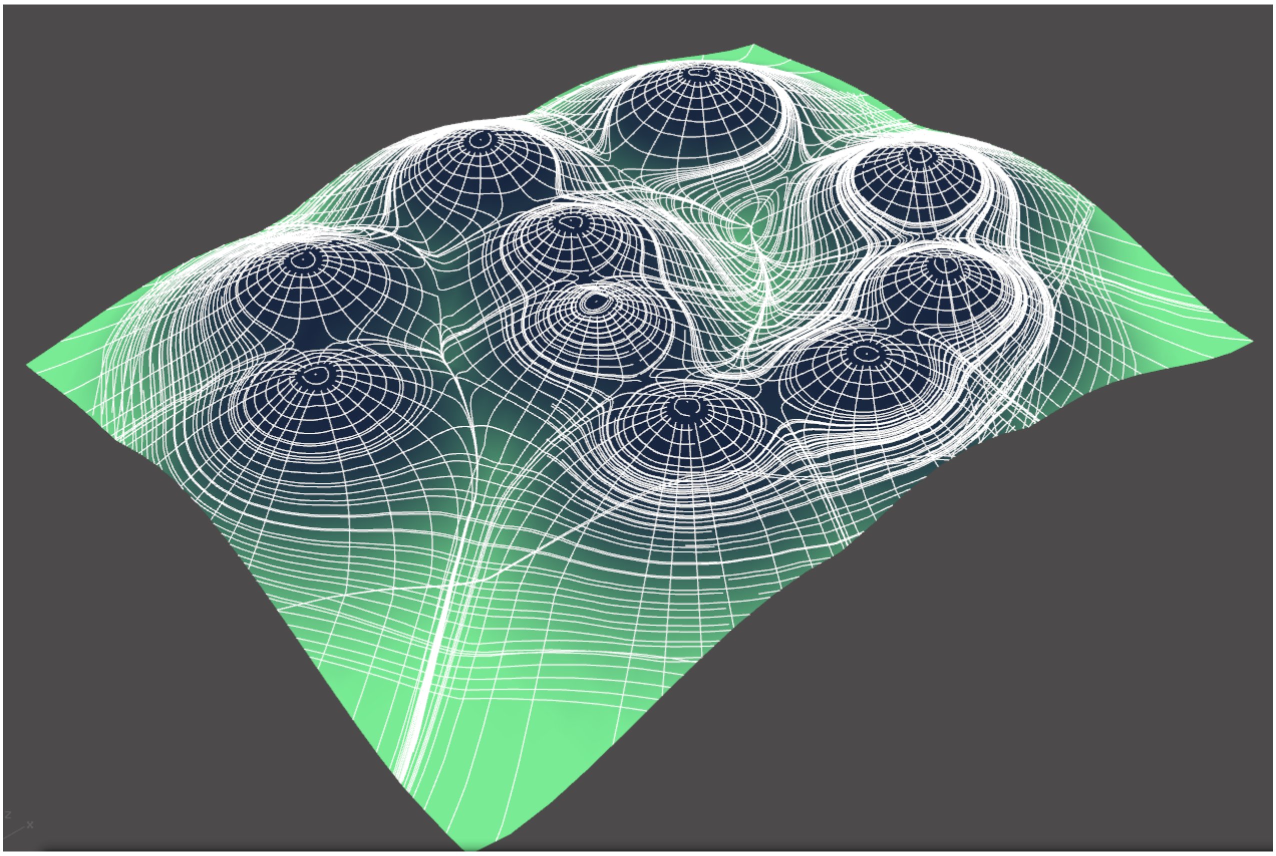

Figure 7. A NURBS surface with equipotential and stream lines texture where point charges coincide with attraction points

5. Conclusions

The paper presents one possible application of design tools based on two families of curves extracted from vector fields in combination with a curved NURBS surface, which is the support for the created grid. This research focuses on the explicit relationship between surface and the texture which describes the surface in an efficient manner. This was accomplished by using the power of parametric design for generating the NURBS surface, and using the geometric qualities of the respective surface in order to generate a correspondence between the surface and the generated grid. This might be a sustainable approach for design experimentation using computational tools as well for rationalization methods of surfaces for fabrication purposes.

6. References

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[3] Idem,pp.18.

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