

## **Study on the Calculation of Beam-walls Subjected to Their Own Weight Action**

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### **Abstract**

The wall-beams, structural elements currently encountered within the civil, socio-cultural and industrial constructions, have -as main charges alongside with their own weight- the loadings uniformly distributed to the inferior part and/or superior part. In this article, the authors propose a new method of solving two types of beam-walls problems, using an analytical method (through using new forms of stress functions/Airy functions): beam-walls simply supported and uninterrupted beam-walls, with equal openings, supported on wide carriers. The obtained results are compared (for validation) with the results provided by the numerical analysis, by using the Finite Element Method.

### **Rezumat**

*Prin acest articol, autorii prezintă o nouă modalitate de rezolvare printr-o metodă analitică (prin utilizarea unor noi forme de funcții ale tensiunilor / funcții Airy), a două tipuri de grinzi-pereți: grinda-perete simplu rezemată și grinda-perete continuă, cu deschideri egale, rezemată pe reazeme late. Rezultatele obținute sunt comparate (pentru validare) cu rezultatele furnizate de către analiza numerică, prin utilizarea Metodei Elementelor Finite.*

**Keywords:** grinda-perete, funcția tensiunilor (funcția Airy), Metoda Elementelor Finite, analiza grinzilor-pereți; beam-walls; stress function (Airy), Finite Element Method, analysis of beam-walls.

### **1. Overall Features**

Wall-beams are frequently encountered in structuring the civil and socio-cultural and industrial constructions. It refers to the reinforced concrete walls of a building that does not constantly support on foundations, the full monolith or prefabricated diaphragms (large panels) of the buildings, horizontally charged panels, about the bunker and silages walls, as well as about the marginal beams of the cylindrical or prismatic covers, and also about the inferior walls of the cooling towers made of curved rotating boards.

One of the main loads is their own weight, a permanent action that is always part of the possible loads. Usually, the loads are applied on contours, in static equilibrium (the case of beam-walls with one opening), or at the inferior part, respectively superior, in the case of continuous beam-walls (also in equilibrium, corresponding to one opening). The own weight is a load continuously and uniformly distributed in the planes parallel with the median plane.

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This action generates a plane stress, possible to be determined by using the methods of Elasticity Theory, given the fact that the report between the height and opening is  $\frac{H}{L} \geq \frac{1}{5}$ .

Slight information exists in the technical literature regarding the stress state produced within the beam-walls from its weight action. Due to this reason, the simple or constantly leaned beam-walls, with equal openings, leaned on wide supports are studied.

Generally, this problem is reduced to determining the stress function (Airy, 1862),  $F(x, y)$ , which must be bi-harmonically

$$\nabla^4 F = \frac{\partial^4 F}{\partial x^4} + 2 \frac{\partial^4 F}{\partial x^2 \partial y^2} + \frac{\partial^4 F}{\partial y^4}, \tag{1}$$

where the stress state is defined by the well-known equations

$$\sigma_x = \frac{\partial^2 F}{\partial y^2}; \quad \sigma_y = \frac{\partial^2 F}{\partial x^2}; \quad \tau_{xy} = \tau_{yx} = -\frac{\partial^2 F}{\partial x \partial y} - Yx - Xy, \tag{2}$$

where X and Y are component after the axes of the continuously distributed charge.

## 2. Beam-wall Simply Supported

Let's consider the beam-wall simply supported, having the following dimensions:  $2a \times 2b$  and thickness  $\delta = 1$  (Fig. 1.a), subjected to its own weight  $X = 0; Y = \gamma$ , where  $\gamma$  represents the weight specific to the constitutive material. In this case, the stress function expressed in the form of the bi-harmonic polynomial is "appropriate" (1)

$$F(x, y) = \frac{\gamma}{4} \left[ \left( 1 - \frac{y^2}{b^2} \right) x^2 y + \left( \frac{a^2}{b^2} - \frac{2}{5} \right) y^3 + \frac{y^5}{5b^2} \right], \tag{3}$$

by means of which the stress state is determined (2):

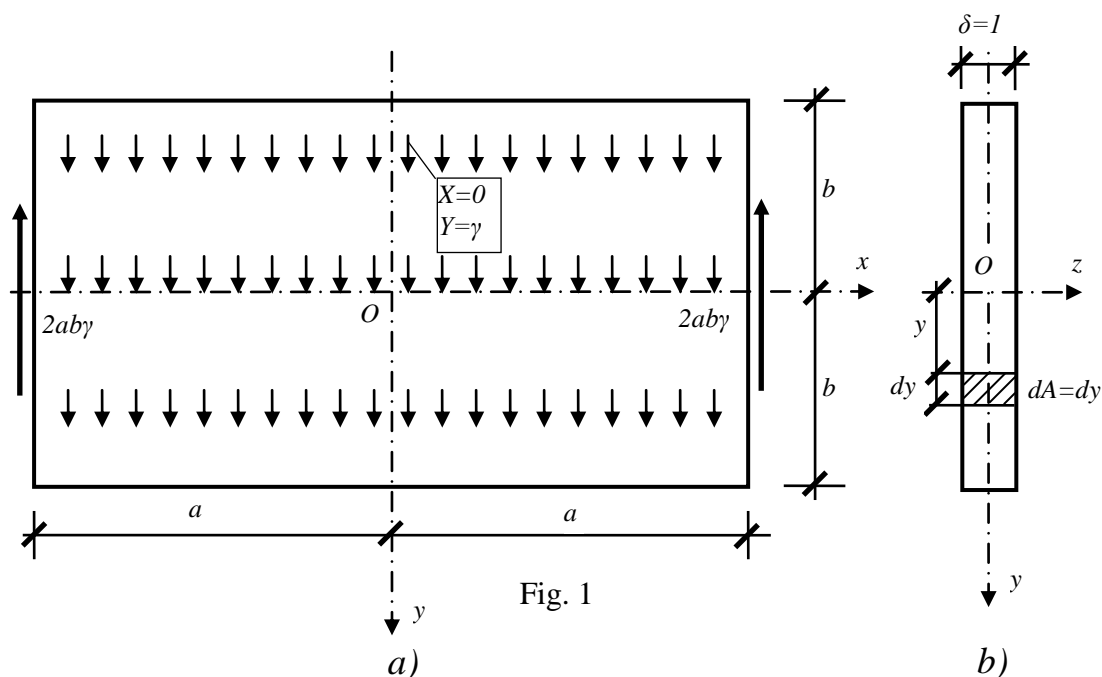


Fig. 1

$$\sigma_x = \frac{\partial^2 F}{\partial y^2} = \gamma b \left[ \frac{3}{2} \frac{a^2}{b^2} \left( 1 - \frac{x^2}{a^2} \right) + \frac{y^2}{b^2} - \frac{3}{5} \right] \frac{y}{b} ;$$

$$\sigma_y = \frac{\partial^2 F}{\partial x^2} = \frac{\gamma b}{2} \left( 1 - \frac{y^2}{b^2} \right) \frac{y}{b} ; \tag{4}$$

$$\tau_{xy} = \tau_{yx} = -\frac{\partial^2 F}{\partial x \partial y} - \gamma x = -\frac{3}{2} \gamma a \left( 1 - \frac{y^2}{b^2} \right) \frac{x}{a} ,$$

which satisfies the outline conditions (Fig.1.a,b):

$$x = \pm a ; N_x = \int_{-b}^{+b} \sigma_x \cdot dy = 0, M_z = \int_{-b}^{+b} \sigma_x \cdot y \cdot dy = 0, T_y = \int_{-b}^{+b} \tau_{xy} \cdot dy = \mp 2ab\gamma ;$$

$$y = \pm b ; \sigma_y = 0, \tau_{xy} = \tau_{yx} = 0. \tag{5}$$

From the mathematical point of view, the problem is equivalent to the situation of loading the beam-wall at the inferior part ( $y = b$ ) and the superior part ( $y = -b$ ), with uniformly distributed load of intensity  $\gamma b$  (Fig. 2). The substantial changes only tolerate  $\sigma_y$ , as a consequence of the outline conditions variations, on the sides  $y = \pm b$ .

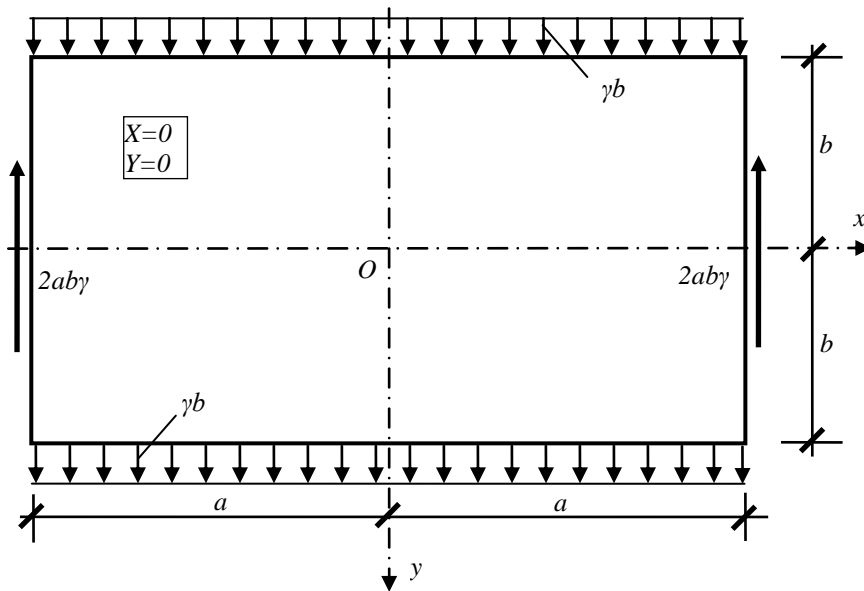


Fig. 2

In this second case, the stress function will have the form of the bi-harmonic polynomial (1)

$$F(x, y) = \frac{\gamma}{4} \left[ \left( 1 - \frac{y^2}{b^2} \right) x^2 y + \left( \frac{a^2}{b^2} - \frac{2}{5} \right) y^3 + \frac{y^5}{5b^2} \right] + \frac{\gamma}{2} x^2 y , \tag{6}$$

And, with the components of the loads, the stress state will be (2)

$$\begin{aligned}\sigma_x &= \frac{\partial^2 F}{\partial y^2} = \gamma b \left[ \frac{3 a^2}{2 b^2} \left( 1 - \frac{x^2}{a^2} \right) + \frac{y^2}{b^2} - \frac{3}{5} \right] \frac{y}{b} ; \\ \sigma_y &= \frac{\partial^2 F}{\partial x^2} = \frac{\gamma b}{2} \left( 3 - \frac{y^2}{b^2} \right) \frac{y}{b} ; \\ \tau_{xy} = \tau_{yx} &= - \frac{\partial^2 F}{\partial x \partial y} = - \frac{3}{2} \gamma a \left( 1 - \frac{y^2}{b^2} \right) \frac{x}{a} ,\end{aligned}\tag{7}$$

Satisfying the outline conditions (Fig. 2):

$$\begin{aligned}x = \pm a ; N_x = 0 , M_z = 0 , T_y = \mp 2 a b \gamma ; \\ y = \pm b ; \sigma_y = \pm \gamma b , \tau_{xy} = \tau_{yx} = 0 .\end{aligned}\tag{8}$$

It may be observed that the stresses  $\sigma_x$  and  $\tau_{xy} = \tau_{yx}$  do not change; if the notation is  $\lambda = \frac{b}{a}$ , at the middle of the beam-wall opening ( $x = 0$ )

$$\sigma_x = \gamma b \left( \frac{3}{2} \frac{1}{\lambda^2} + \frac{y^2}{b^2} - \frac{3}{5} \right) \frac{y}{b} ; \tau_{xy} = \tau_{yx} = 0\tag{9a}$$

and on the support ( $x = \pm a$ )

$$\sigma_x = \gamma b \left( \frac{y^2}{b^2} - \frac{3}{5} \right) \frac{y}{b} ; \tau_{xy} = \tau_{yx} = \mp \frac{3}{2 \lambda} \left( 1 - \frac{y^2}{b^2} \right) \gamma b ,\tag{9b}$$

the self-equilibrated stress  $\sigma_x$  ( $N_x = 0 ; M_z = 0$ ), not depending on  $\lambda$ . On the basis of Barré de Saint-Venant's the principle, this system of „exterior forces” equivalent to zero has only a local effect, negligible if one withdraws from the beam-wall's boundaries.

In table 1, there are calculated the values of stresses  $\sigma_x$ , in the middle of opening the beam-wall ( $x = 0$ ), taking  $\lambda$  into account. For  $\lambda \leq \frac{1}{5}$  (0,2), it results that the stresses  $\sigma_x$  may be deduced using the formula of Navier from the common beams, whilst if  $\lambda > \frac{1}{5}$ , these must be calculated using the Elasticity Theory. As a value,  $\sigma_x$ , decreases along with  $\lambda$ . It is interesting to underline the variation of the stress  $\sigma_x$ , for  $\lambda \geq 5$  (the beam-wall very high compared with the length), the diagram tending to become identical qualitatively as well as quantitative to the one of the support (Table 2). In Figure 3.a,b there are represented the diagramme  $\sigma_x|_{x=0}$  for  $\lambda = 0,50$  and  $\sigma_x|_{x=\pm a}$ , the same for any  $\lambda$ . In table 3, there are calculated the values of  $\tau_{xy}|_{x=a}$  in the function of  $\lambda$  and, in Figure 4, their diagram for  $\lambda = 0,50$  is sketched. In all these cases,  $\tau_{xy \max} = \frac{3 T_y}{2 A} = \frac{3 \gamma b}{2 \lambda}$ , the values decrease inversely with the increase of  $\lambda$ .

The stresses  $\sigma_y$ , quantitative and qualitative different in the two cases (Tabelul 4; Fig. 5.a, b), do

not depend of  $x$  or  $\lambda$ , their symbols being defined by the relations (4) and (7):  $\sigma_y = \frac{\gamma b}{2} \left( 1 - \frac{y^2}{b^2} \right) \frac{y}{b}$

and, respectively,  $\sigma_y = \frac{\gamma b}{2} \left( 3 - \frac{y^2}{b^2} \right) \frac{y}{b}$ .

$$\sigma_x|_{x=0}$$

$$\sigma_x|_{x=\pm a}$$

Table 1

Table 2

$\lambda = b/a$	0,1	0,2	0,5	1,0	1,5	2,0	5,0
$y/b$							
1,00	150,400	37,900	7,600	1,900	1,067	0,775	0,460
0,50	74,825	18,575	2,825	0,575	0,158	0,012	-0,145
0,00	0,000	0,000	0,000	0,000	0,000	0,000	0,000
-0,50	-74,825	-18,575	-2,825	-0,575	-0,158	-0,012	0,145
-1,00	-150,400	-37,900	-7,600	-1,900	-1,067	-0,775	-0,460
Navier	$\pm 150,000$	$\pm 37,500$	$\pm 6,000$	$\pm 1,500$	$\pm 0,667$	$\pm 0,375$	$\pm 0,060$
Factor	$\gamma b$						

$y/b$	$\sigma_x$
1,0000	0,400
0,7746	0,000
0,7500	-0,028
0,5000	-0,175
0,2500	-0,134
0,0000	0,000
-0,2500	0,134
-0,5000	0,175
-0,7500	0,028
-0,7746	0,000
-1,0000	-0,400
Factor	$\gamma b$

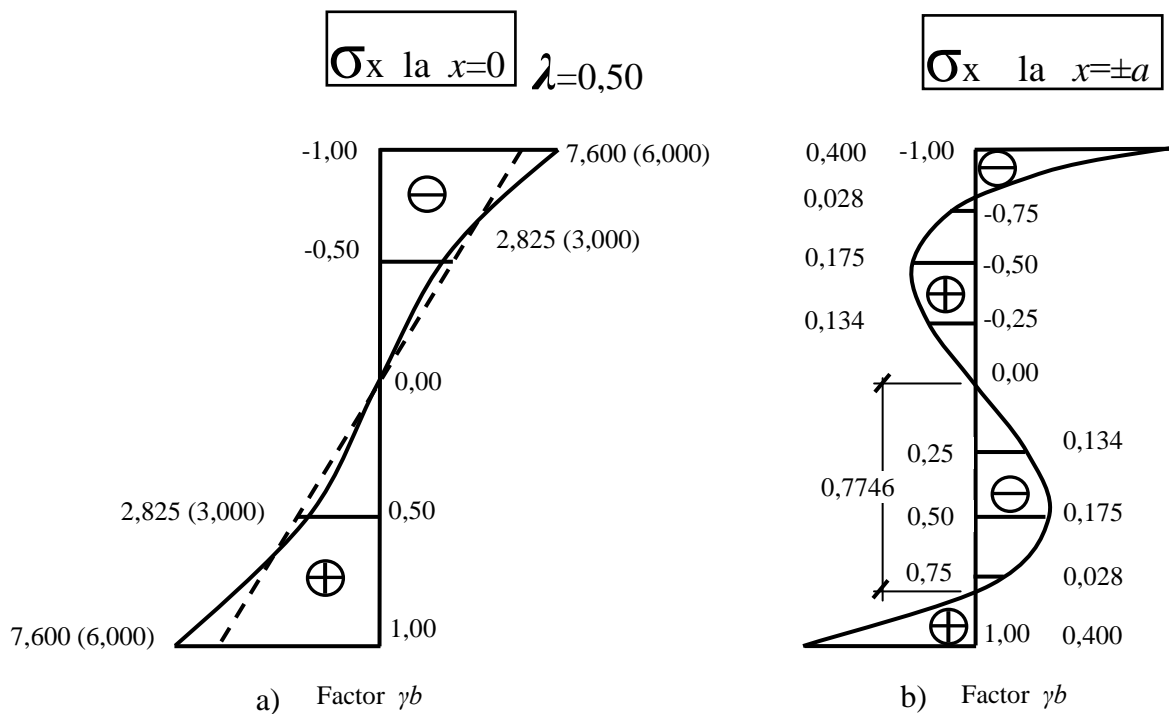


Fig. 3

$$\tau_{xy} |_{x=a}$$

Table 3

$\lambda = b/a$ $y/b$	0,1	0,2	0,5	1,0	1,5	2,0	5,0
1,00	0,000	0,000	0,000	0,000	0,000	0,000	0,000
0,50	-11,250	-5,625	-2,250	-1,125	-0,750	-0,562	-0,225
0,00	-15,000	-7,500	-3,000	-1,500	-1,000	-0,750	-0,300
-0,50	-11,250	-5,625	-2,250	-1,125	-0,750	-0,562	-0,225
-1,00	0,000	0,000	0,000	0,000	0,000	0,000	0,000
Factor	$\gamma b$						

$$\tau_{xy} \text{ la } x=a \quad \lambda=0,50$$

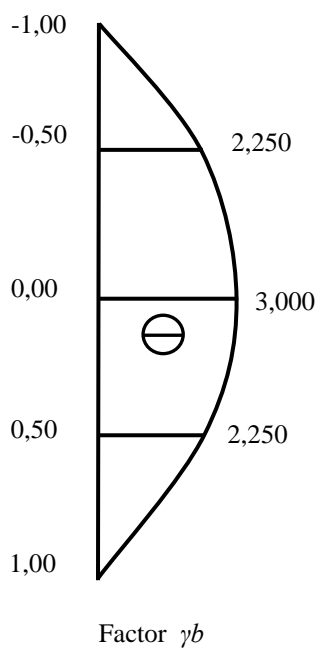


Fig. 4

$$\sigma_y$$

Table 4

$y/b$	$\sigma_y$	
	Primul caz	Al doilea caz
1,00	0,000	1,000
0,75	0,164	0,914
0,50	0,187	0,687
0,25	0,117	0,367
0,00	0,000	0,000
-0,25	-0,117	-0,367
-0,50	-0,187	-0,687
-0,75	-0,164	-0,914
-1,00	0,000	-1,000
Factor	$\gamma b$	

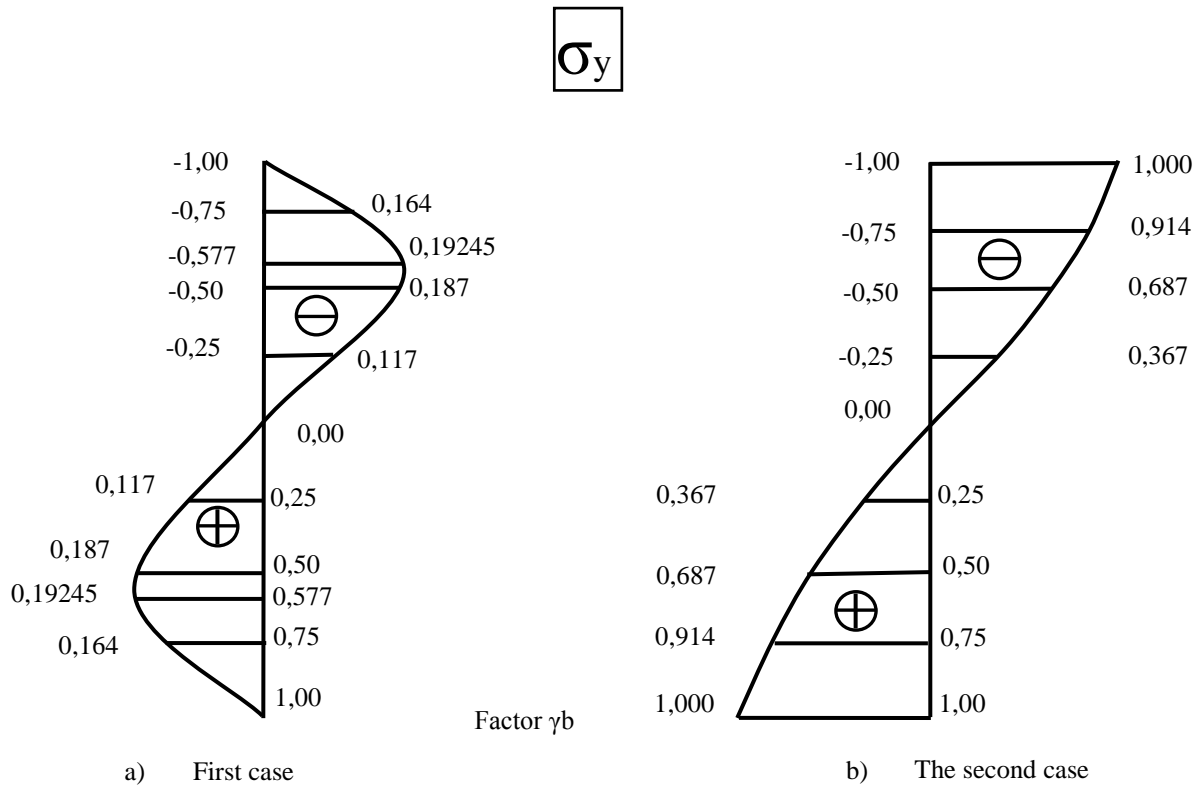


Fig. 5

The reinforcement of the beam-walls constitutes a main problem in their own structure. Usually, there are two ways of reinforcement: by using *orthogonal networks*, which are easier to perform, but with larger use of steel, and *trajectory reinforcement*, in which the reinforcement follows the main directions of the stretch stresses; in a simplified manner, horizontally, at the part that is stretched to the opening, and bended, in the area of the supports (perpendicular on the cleaves), but as the height of the beam-wall rises, the cleaves tend to become vertical (Fig. 6). The normal stress of stretching  $\sigma_y$  (whichever is their distribution on height), that are coming from the action of “attached loads”, will be taken from the vertical berthing reinforcements [2].

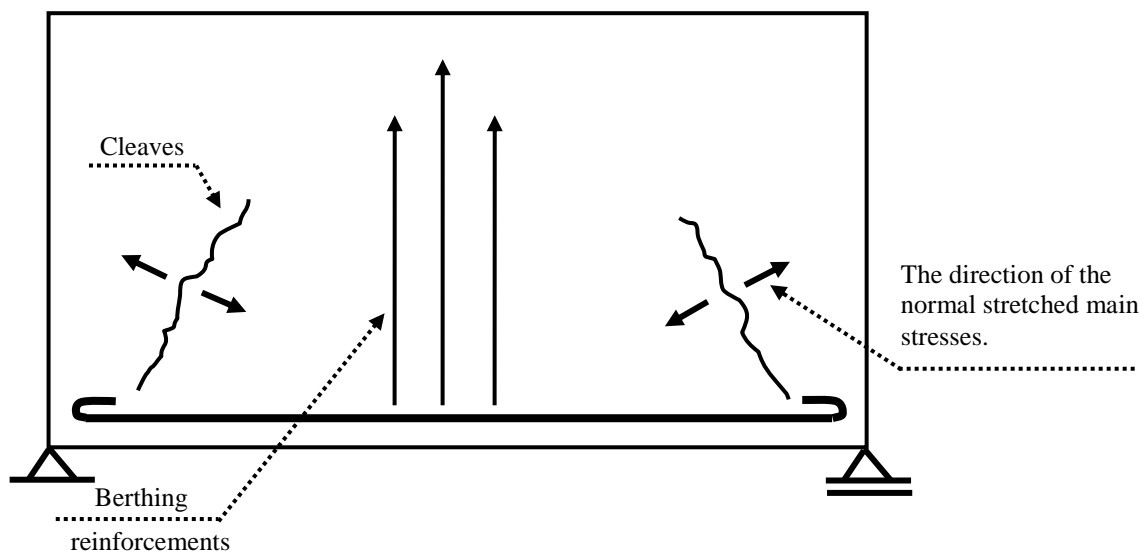


Fig. 6

### 3. The Continuous Beam-wall of Equal Openings, Leaning on Wide Supports

Let's consider the continuous beam-wall presented in Figure 7, taking into account the fact that its own weight operates on the inferior part  $p = 2\gamma b$ , and the reaction in the wide supports ( $2c$ ) is  $p_1 = \frac{2\gamma b}{\varepsilon}$ , where  $\varepsilon = \frac{c}{a}$ . The stress in the beam-wall, where  $\alpha_n = \frac{n\pi}{b}$  may be obtained with the relations (2.64) from [3], in which  $p$  is replaced with  $2\gamma b$  and  $h$  with  $b$ .

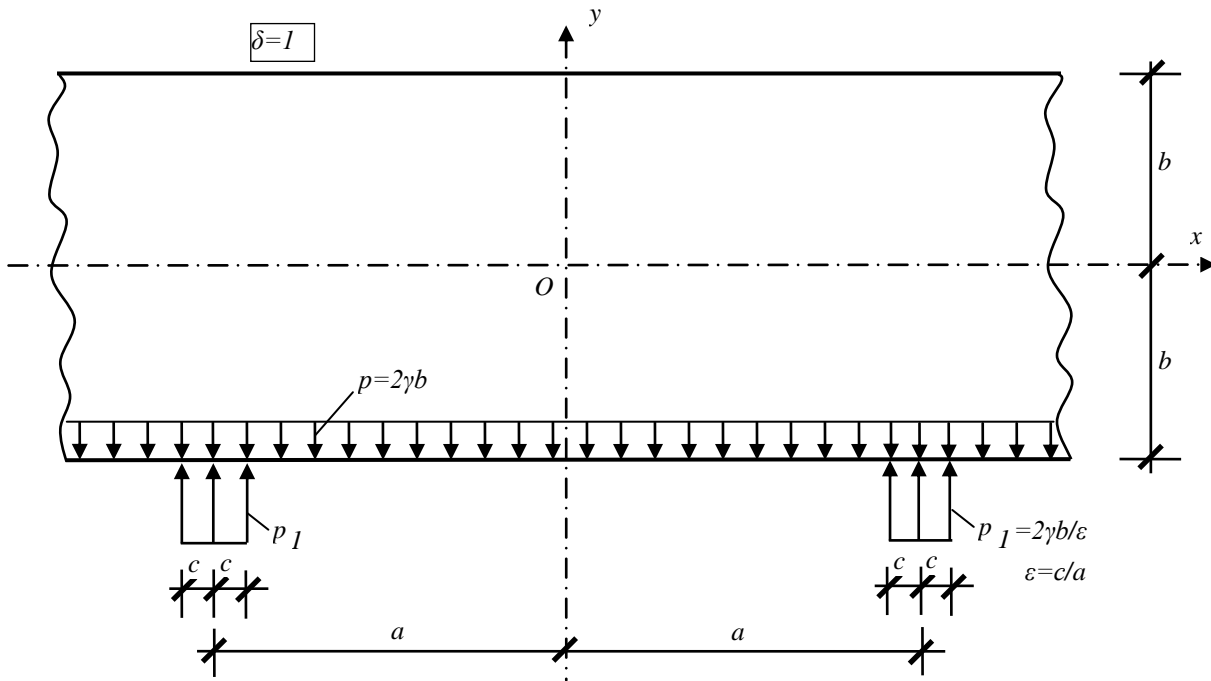


Fig. 7

$$\begin{aligned} \sigma_x &= \frac{2\gamma b}{\pi\varepsilon} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \left[ \frac{f_1(\alpha_n y)}{g_1(\alpha_n b)} - \frac{f_2(\alpha_n y)}{g_2(\alpha_n b)} \right] \cdot \sin(n\pi\varepsilon) \cdot \cos(\alpha_n x) ; \\ \sigma_y &= \frac{2\gamma b}{\pi\varepsilon} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \left[ \frac{f_3(\alpha_n y)}{g_1(\alpha_n b)} - \frac{f_4(\alpha_n y)}{g_1(\alpha_n b)} \right] \cdot \sin(n\pi\varepsilon) \cdot \cos(\alpha_n x) ; \\ \tau_{xy} = \tau_{yx} &= \frac{2\gamma b}{\pi\varepsilon} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \left[ \frac{f_5(\alpha_n y)}{g_1(\alpha_n b)} - \frac{f_6(\alpha_n y)}{g_2(\alpha_n b)} \right] \cdot \sin(n\pi\varepsilon) \cdot \sin(\alpha_n x) , \end{aligned} \quad (10)$$

whilst the contour conditions are satisfied

$$\begin{aligned} y = b & ; & \sigma_y = 0, \quad \tau_{xy} = \tau_{yx} = 0 \\ y = -b & ; & \sigma_y = p = 2\gamma b \quad (\text{întindere, } x = 0) , \\ & & \sigma_y = -p_1 + p = -\frac{2\gamma b}{\varepsilon}(1 - \varepsilon) \quad (\text{compresiune, } x = \pm a) , \\ & & \tau_{xy} = \tau_{yx} = 0 . \end{aligned} \quad (11)$$

The functions  $f_1(\alpha_n y)$ ,  $f_2(\alpha_n y)$ ,  $f_3(\alpha_n y)$ ,  $f_4(\alpha_n y)$ ,  $f_5(\alpha_n y)$ ,  $f_6(\alpha_n y)$  and  $g_1(\alpha_n b) = f_3(\alpha_n b)$ ,



$g_2(\alpha_n b) = f_4(\alpha_n b)$  are presented in Table [1], for different reports  $\lambda = \frac{b}{a} \left( \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, 1, \frac{4}{3}, \frac{3}{2}, 2 \right)$ .

In the tables 5 and 6 there are presented the stress values  $\sigma_x$  și  $\sigma_y$  at the middle of the opening ( $x = 0$ ) and in the axis of the supports ( $x = \pm a$ ), in case  $\lambda = 0,50$  și  $\varepsilon = 0,10$ , where the first value of the series is marked in the table. The variation diagrams are presented in Fig. 8 and Fig. 9.

$\sigma_x$

Table 5

$y/b$	$\sigma_x$	
	$x = 0$	$x = \pm a$
1,00	-0,532	0,625
0,75	-0,329	0,380
0,50	-0,208	0,271
0,25	-0,125	0,231
0,00	-0,047	0,232
-0,25	0,052	0,243
-0,50	0,191	0,197
-0,75	0,392	-0,222
-1,00	0,656	-4,658
Factor	$4\gamma b$	

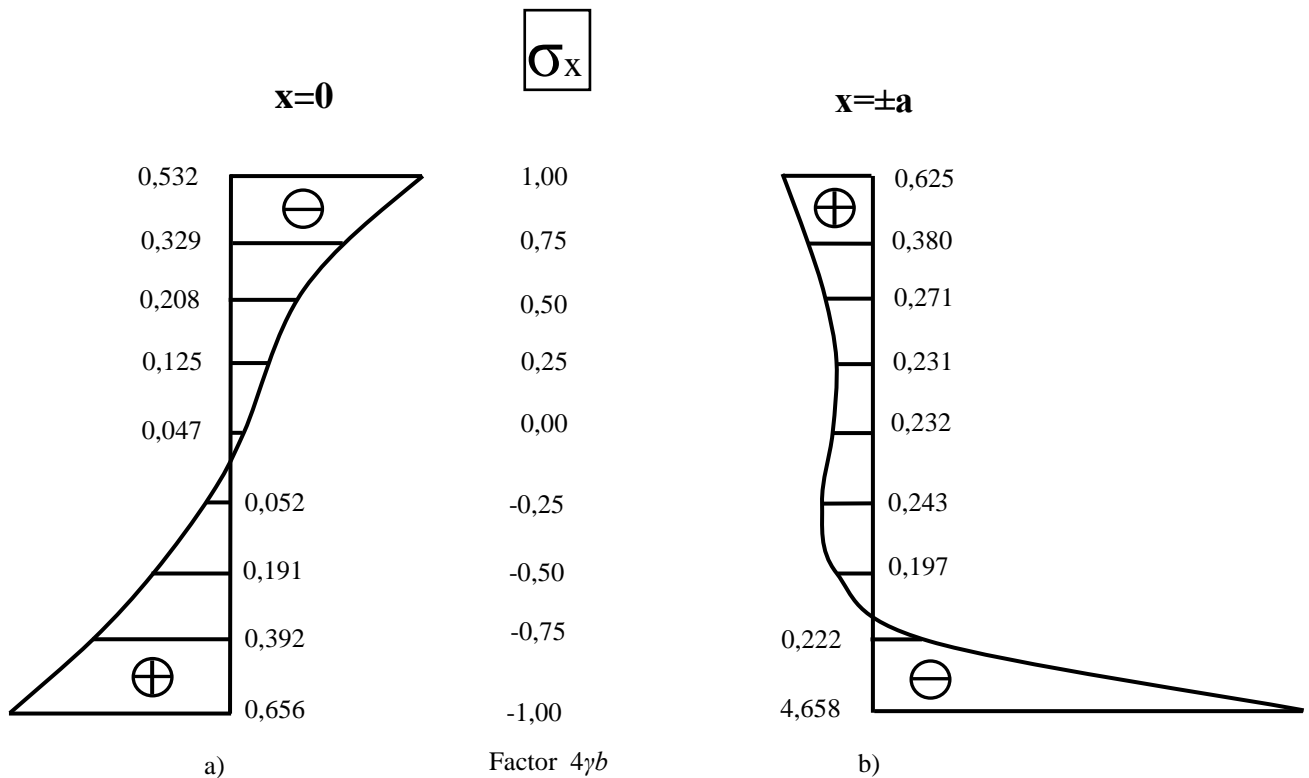


Fig. 8

$$\sigma_y$$

Tabelul 6

$y/b$	$\sigma_y$	
	$x=0$	$x=\pm a$
1,00	0,000	0,000
0,75	0,022	-0,199
0,50	0,079	-0,713
0,25	0,177	-1,596
0,00	0,345	-3,102
-0,25	0,621	-5,595
-0,50	1,049	-9,445
-0,75	1,600	-14,406
-1,00	1,967	-17,704
Factor	$2\gamma b$	

$$\sigma_y$$

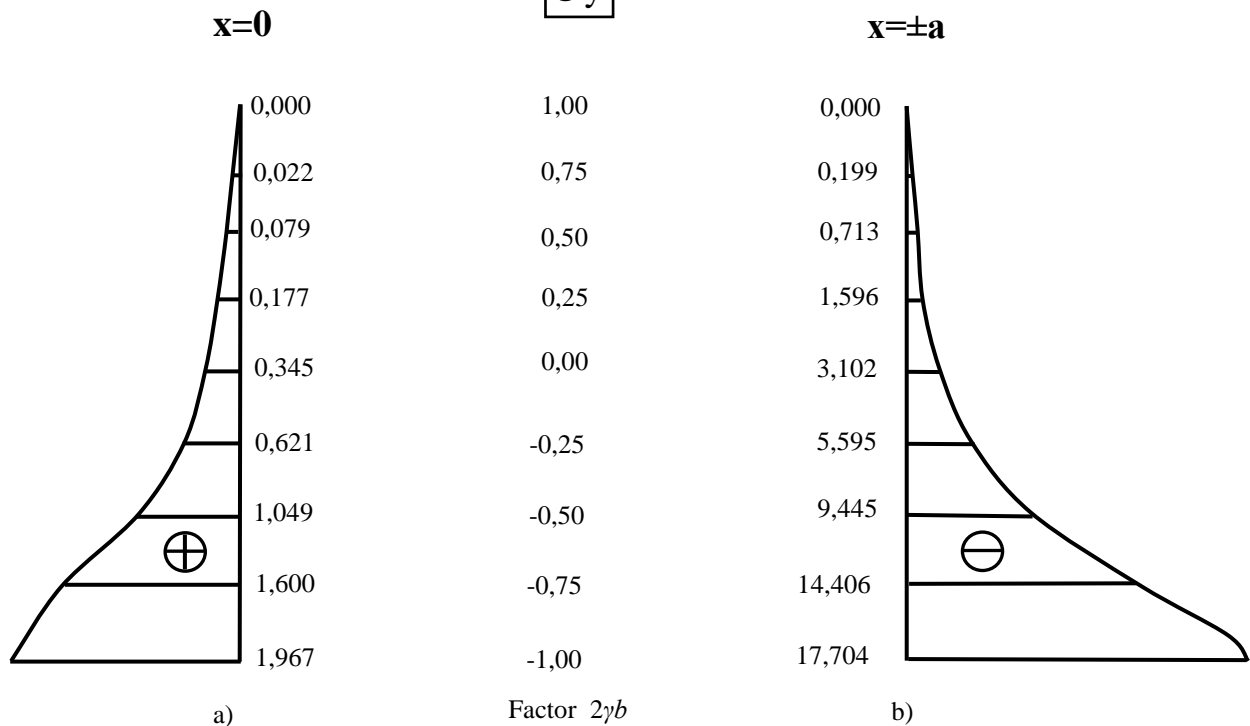


Fig. 9

#### 4. The Analysis Using the Finite Element Method

In order to verify the obtained results by using the stress functions (Airy) presented above, there have been generated numerical analysis by using the designs operated in „Autodesk ROBOT Structural Analysis 2014”.

The present work further presents the obtained results in only three cases of analysed problems.

##### 4.1 The Beam-Wall with One Opening Simply Supported, Being Under the Action of its Own Weight.

The geometrical dimensions of the beam-wall:  $a = 2,0m$ ,  $b = 1,0m$  and  $\delta = 0,1m$  (where  $\lambda = 0,50$

and  $\varepsilon = 0,10$ ); the specific weight of the material  $\gamma = 25\text{ kN/m}^3$ .

Due to the different design of the supports, only the values of the stresses  $\sigma_x$  at the middle of the opening ( $x = 0$ ) may be compared:

1. Theoretically,  $\sigma_{x_{\max}} = 7,60 \cdot 25 \cdot 10^{-2} = 1,90\text{ daN/cm}^2$  after Navier,  $1,50\text{ daN/cm}^2$ );
2. By using Finite Element Method  $\sigma_{x_{\max}} = 1,62\text{ daN/cm}^2$ ; lower value, therefore, with 14,73%.

#### 4.2 The Beam-Wall with One Opening, Simply Supported, Being Under the Action of a Force Evenly Distributed at the Superior and Inferior Parts

The geometrical dimensions of the beam-wall  $a = 2,0\text{ m}$ ,  $b = 1,0\text{ m}$  and  $\delta = 0,1\text{ m}$ , (where  $\lambda = 0,50$  and  $\varepsilon = 0,10$ ); the charge evenly distributed at the superior and inferior part  $\gamma b = 25\text{ kN/m}^2$ .

In this case, in the middle of the opening ( $x = 0$ ):

1. Theoretically  $\sigma_{x_{\max}} = 1,900\text{ daN/cm}^2$
2. By Finite Element Method,  $\sigma_{x_{\max}} = 1,545\text{ daN/cm}^2$ ; therefore, with lower value, 18,68%.

The stresses  $\sigma_y$  have the following values:

1. Theoretically  $\sigma_{y_{\max}} = 1 \cdot 25 \cdot 10^{-2} = 0,25\text{ daN/cm}^2$
2. By Finite Element Method  $\sigma_{y_{\max}} = 0,260\text{ daN/cm}^2$  (almost identical), not depending on  $\lambda$ .

#### 4.3 The Continuous Beam-wall, of Equal Openings, Leaned on Wide Supports, Being Under the Action of a Charge Evenly Distributed at the Inferior Part.

The geometrical dimensions of the beam-wall:  $a = 2,0\text{ m}$ ,  $b = 1,0\text{ m}$ ,  $c = 0,2\text{ m}$  and  $\delta = 0,1\text{ m}$ , (where  $\lambda = 0,50$  and  $\varepsilon = 0,10$ ); the charging evenly distributed to the inferior face  $2\gamma b = 50\text{ kN/m}^2$ .

Comparatively, the results are listed in Table 7.

Table 7

		$\sigma_x$ ( $\text{daN/cm}^2$ )		$\sigma_y$ ( $\text{daN/cm}^2$ )	
		$x = 0$	$x = \pm a$	$x = 0$	$x = \pm a$
Analytic	$y = -b$	+0,656	-4,658	+0,984	-8,852
	$y = +b$	-0,532	+0,625	0,000	0,000
MEF	$y = -b$	+0,728	-5,690	+1,029	-9,450
	$y = +b$	-0,513	+0,641	-0,004	+0,005

A good sequence of the results may be observed. In the MEF case, the stresses  $\sigma_x$  and  $\sigma_y$  have higher values; therefore, where  $x = 0$  (the middle of the opening), for the inferior part of the beam-wall, the stretching stresses  $\sigma_x$  are in the report  $0,728/0,656 = 1,109$  and the stretching stresses  $\sigma_y$  in the report  $1,029/0,984 = 1,045$ , and, at  $x = \pm a$  (on the support), the compression stresses  $\sigma_x$  și  $\sigma_y$  are in the reports  $5,690/4,658 = 1,221$ , and, respectively  $9,450/8,852 = 1,068$ .

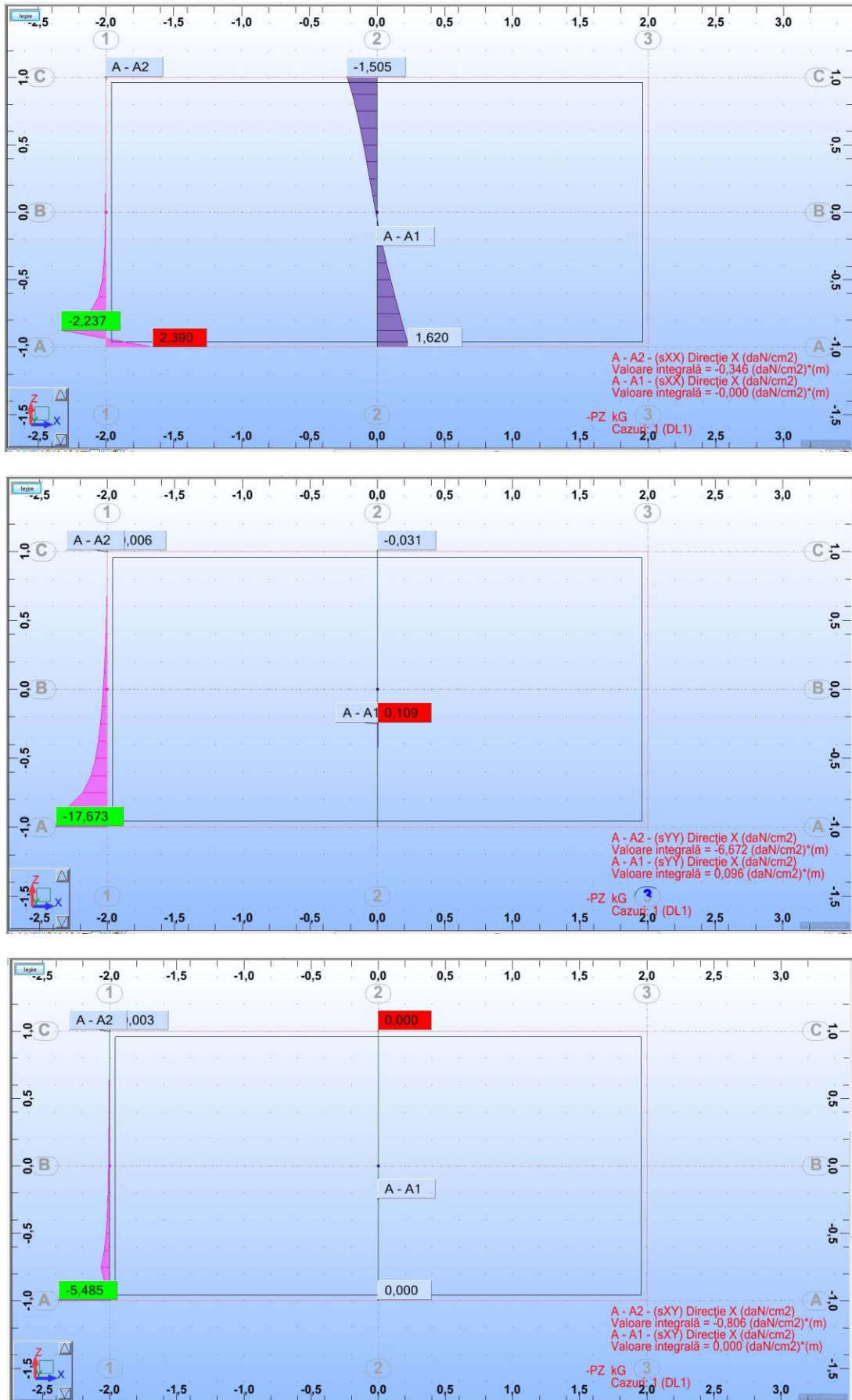


Fig. 10 (a, b, c)

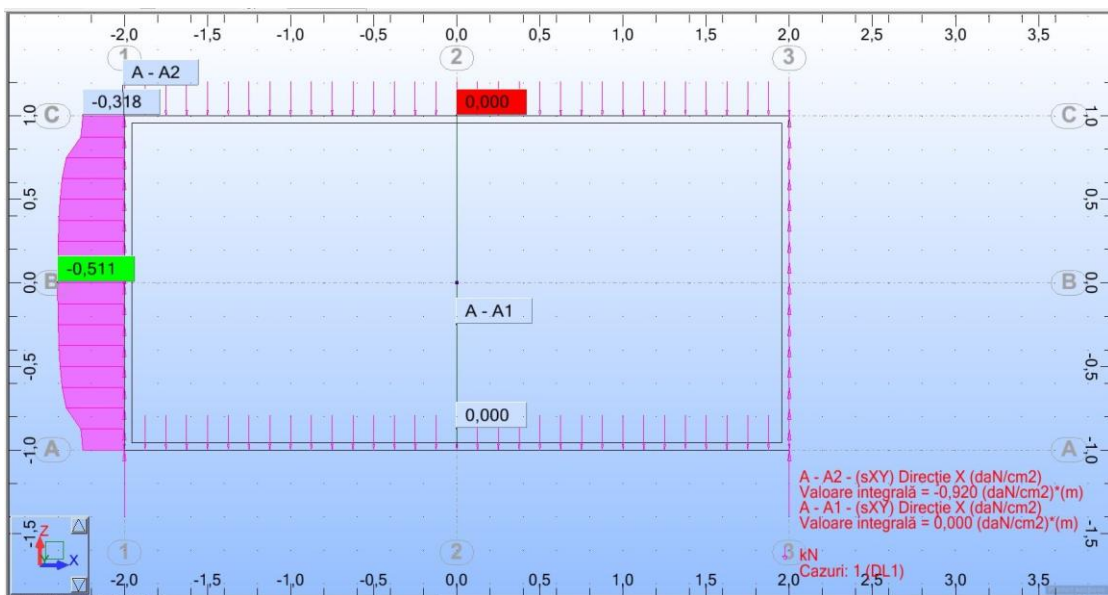
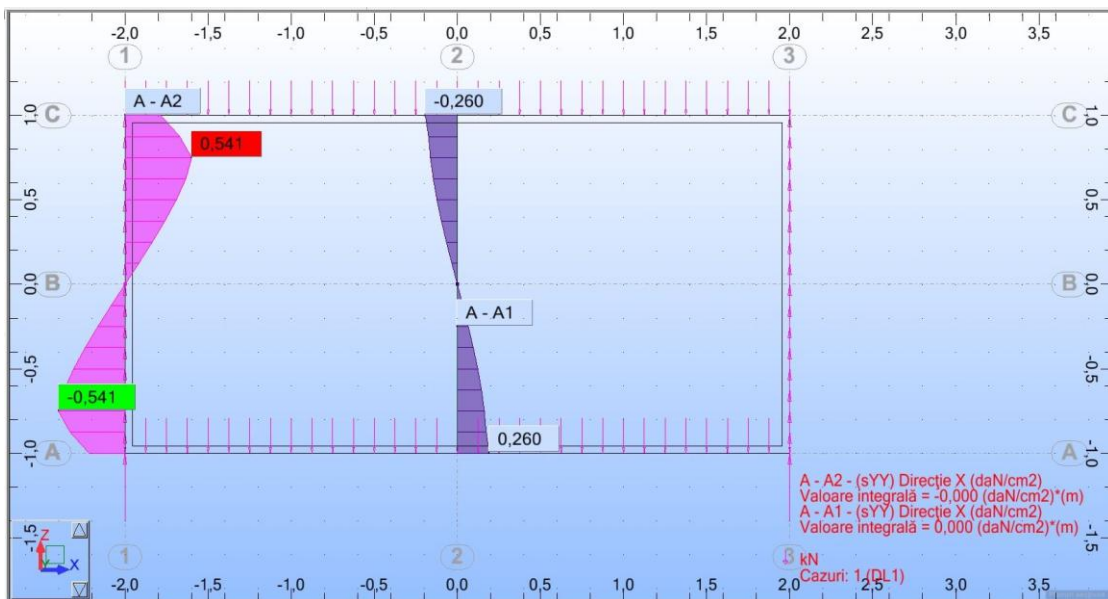
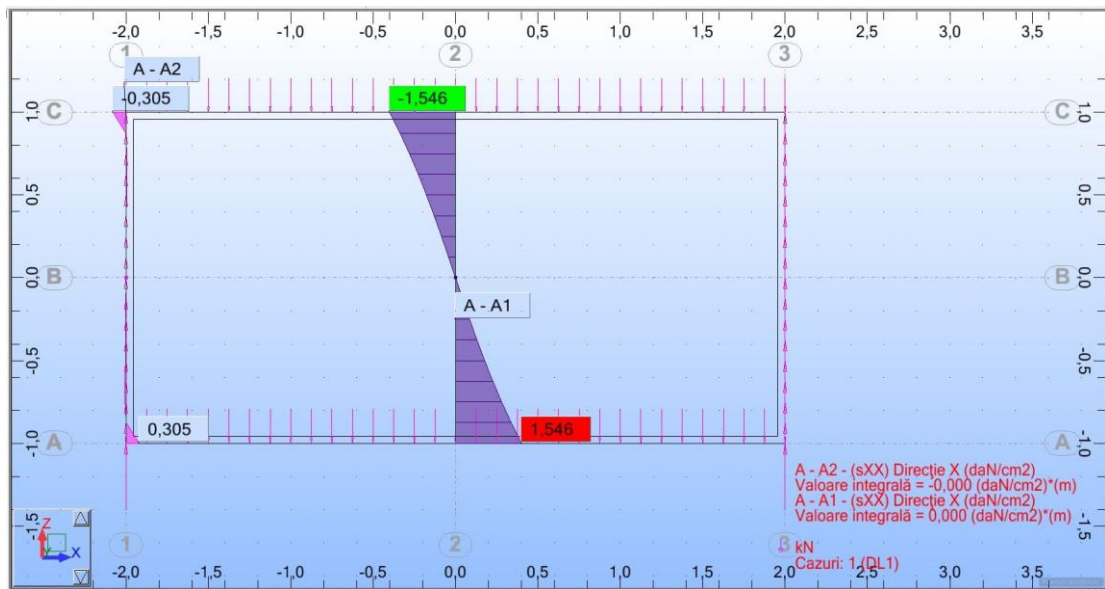


Fig. 11 (a, b, c)

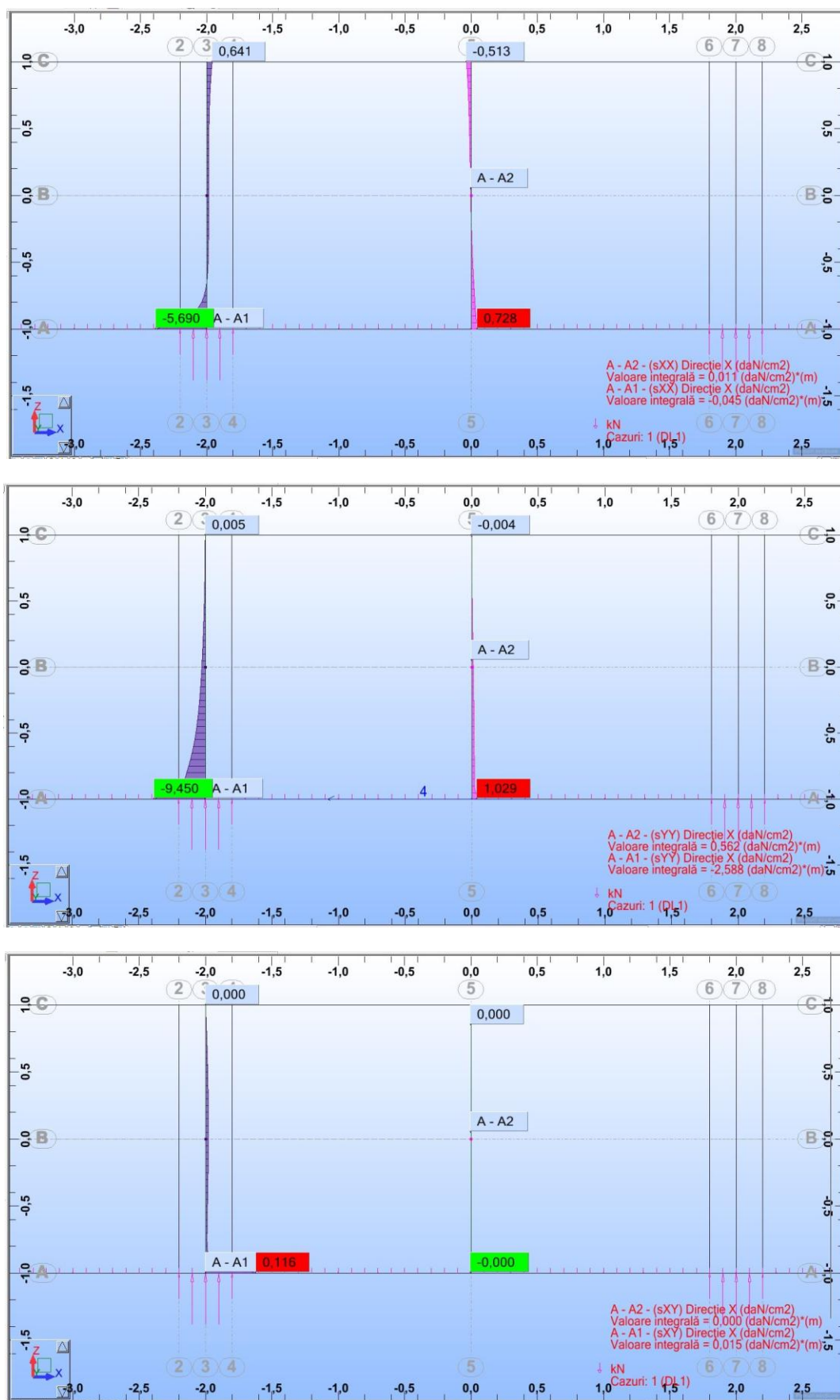


Fig. 12 (a, b, c)

## 5. Conclusions

The results obtained from the analysis performed by using the program „Autodesk ROBOT Structural Analysis 2014”, are presented in Fig. 10÷12 as diagrams of the stresses  $\sigma_x$ ,  $\sigma_y$  și  $\tau_{xy}$  in the fields sections and on the support of the studied beam-walls, emphasize the fact that the states of stress obtained by using Finite Element Method are very close to the ones obtained by using stress functions (Airy) proposed in the paragraphs 2 and 3, excepting the supports of the beam-walls simply supported, where the sketching of the reactions are made analytically by self-equilibrated loads.

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