*Acta Technica Napocensis: Civil Engineering & Architecture Vol. 56, No. 2 (2013)*
**Journal homepage:** http://constructii.utcluj.ro/ActaCivilEng
*Special Issue: First International Conference for PhD Students in Civil Engineering, CE-PhD 2012.*

# Generalization of the Novel, Ayrton-Perry Formula Based Method for Lateral-Torsional Buckling of Members

Bettina Badari[*1], Ferenc Papp[2]

[1,2] *Budapest University of Technology and Economics, Dept. of Structural Engineering, 3-5 Műegyetem rkp., 1111, Budapest, Hungary*

*(Accepted 28 August 2013; Published online 30 September 2013)*

## Abstract

*Many theses have been published on the now much-discussed phenomenon of lateral-torsional (LT) buckling of beams with the aim of solving the problems of design formulae given by the EN 1993-1-1. In this paper a novel Ayrton-Perry formula based method is introduced for the calculation of LT buckling resistance of beams. According that, the finite element model is detailed which was used for the numerical tests needed to produce the calibration database. For the proper modeling, the consequence of different types of geometrical and material imperfection is examined. The calibration method of the formula is described in this paper for the case of simple beams and a novel method is proposed for the calculation of the LT buckling resistance. The generalization of the novel method is examined for beams with prevented end-warp, loaded by constant moment distribution and beams loaded by triangular moment distribution. Samples are presented for the examined cases.*

**Keywords:** LT buckling, Ayrton-Perry formula, calibration and generalization, imperfection, prevented end-warp, triangular moment-distribution

## 1. Research work on the novel Ayrton-Perry formula

During the analysis of steel structures the determination of the stability resistance is one of the most significant verification since usually the loss of stability is the governing problem. For these complex mechanical behaviors the Eurocode standards endeavor to give simplified methods to make the design process easier. One of the most important simplifications is the principle of member isolation whose widespread application is the effective length method [3]. This method specifies the boundary conditions of a single member through the definition of an effective length which is used to evaluate simple stability equations on a virtual equivalent member. The drawback of this method arises from that the number of possible structural configurations is unlimited, however standard equations can be provided only for specific cases. Therefore, despite the complexity of the interaction factors presented in Eurocode 3, the range of application is very limited [4].

An appropriate solution for the above problem is the Global type Design Approach (GDA) which uses one single model for both the mechanical analysis and for stability design. Through GDA the

[*] Corresponding author: Bettina Badari Tel./ Fax.: +36 30 5889565
E-mail address: badaribetti@gmail.com

same structural model is evaluated using a Global Model based Buckling Analysis (GMBA) instead of doing calculations on a virtual equivalent member, which gives realistic results for the stability problems of the structure. Therefore, the GDA reveals a deeper insight into the structural behavior of the model. Unfortunately, design rules to support the results of the GMBA are not properly developed yet, hence the advantages of the GDA cannot be utilized by the practising engineers. Eurocode 3 allows applying the principles of GDA through the Overall Imperfection Method or the General Stability Method, but the guidance is not comprehensive and includes several restrictions making the application difficult.

Recognizing the demand for a complete, entirely derived and comprehensively verified design method, a new research work started according to the GDA. The aim of the work is the development of a novel stability design process based on the generalized Ayrton-Perry formula. The derivation of the equations of the formula is introduced in [1] for the case of the LT buckling of simple, prismatic beams with I-shaped sections, end-fork, free to warp boundary conditions, subjected to pure bending (hereinafter basic model). As the first step of the research it was examined that the formulae determined in [1] how can be calibrated and a beneficial, novel method with the calibrated formulae can be proposed or not. For the results of an initial, representative test program it was proved that the Ayrton-Perry formula is appropriate and has many advantages for the case of simple beams [2]. In further work the calibration process of the formula was carried out for an extended test program and a novel method was proposed for the stability design of the basic model [5].

In this paper the novel, calibrated Ayrton-Perry formula based method for LT buckling of simple beams is presented and the generalization is examined. The subjects of the examination are the beams with boundary conditions different from the basic model (namely end-fork with prevented end-warp) and the beams with triangular moment distribution. In the paper the proposed method is detailed for the three examined cases through examples.

## 2. Finite element model for numerical tests

For the examinations and the calibration GMNI calculations of tested members were carried out to determine the needed LT buckling resistances. The numerical simulations of beams with different load distributions and boundary conditions were carried out by shell finite element models, in ANSYS software. The steel members were modeled with simplified cross-sections shown in Fig. 1-a, which were equivalent with the hot-rolled I-sections chosen for the tests. The geometry of the cross-sections and the length of the members were given by input parameters for the model creation. At the ends of the beams specific models of supports were defined, where every nodes of each cross-sections were connected to a master node, see in Fig. 1-a. With this construction the boundary conditions of warp and different types of supports could be specified on one node, and at the same time the numerical errors arising from the concentrated conditions could be avoided. The force and moment type loads of beams were defined in form of stress on the lines of end-sections. Material and geometrical imperfections were modeled on members, which will be detailed later.

The models were constructed with 4-node, SHELL181 type finite strain shell elements, which can model the nonlinear behavior. Examinations were performed to determine the effect of changing the size of longitudinal and transverse finite element mesh on the result of numerical simulations. Based on the findings of the tests, division into 100 parts was chosen along the members to get properly accurate numerical simulations with optimal computational time. However these aspects did not justify, the accurate modeling of residual stresses needed small size finite element mesh in transverse direction. Therefore, along the half of flanges and also the web division into 6 parts was chosen, shown in Fig. 1-a. The material behavior of the beams was modeled with linear elasticideally plastic material model with $E = 210GPa$ Young-modulus and yield criterion belonging to the standard yield strength of the material grade.

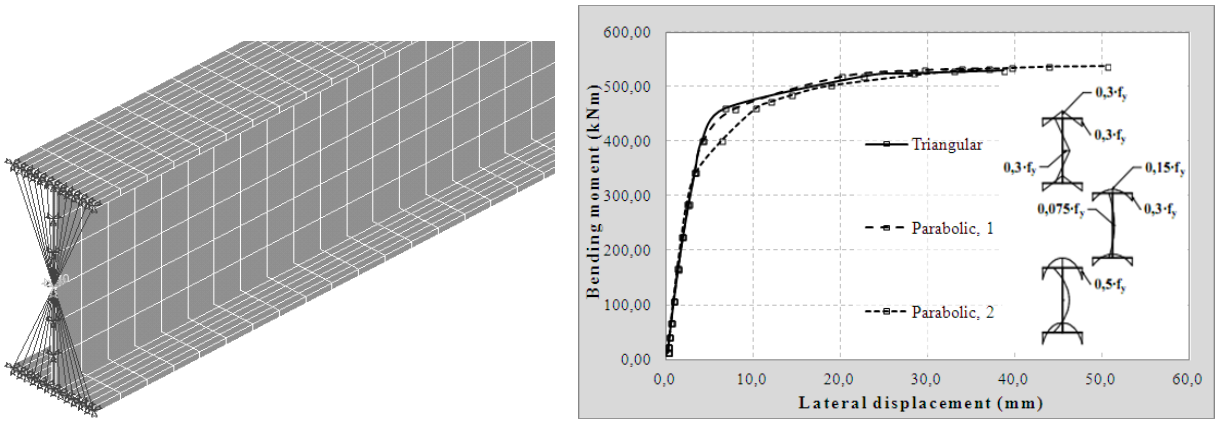



Figure 1. a- FEM model of beams; b - M-v curves of HEB300 with different residual stress models

To model the residual stresses of the members, mainly two types of distribution can be found in use at the international research works. These are the triangular distribution on the flanges and also the web (e. g. in [7]) and the parabolic distribution (e. g. in [8]). Boissonnade et al. carried out numerical simulations to examine the differences between the LT buckling resistances of beams with different residual stress distributions for the case of end-fork boundary conditions [8]. Similarly to this research, we carried out numerical simulations on beams with prevented end-warp conditions to prove the validity of previous findings in [8]. The Fig. 1-b shows examples for the test results. On the diagram the bending moment is plotted over the lateral displacement of the midpoint of upper flange, hereinafter the M-v curves of beams with HEB300 profile, 8500mm member length and S355 material grade are shown. In Fig. 1-b the solid line belongs to the results of member with triangular distribution, long-dashed line to the parabolic used in [8], and short-dashed line to the parabolic determined in [6]. Based on the results it can be stated that the differences between the behavior of beams with different residual stress distributions are negligible. The largest difference between the ultimate bearing capacities of beams presented as examples in Fig. 1-b was less than 2%. In further numerical tests the parabolic residual stress distribution determined in [6] was used with amplitude depending on the geometry of the cross-section: if the height/width (h/b) ratio of the profile is over 1,2 the maximum value of residual stress at the top of flanges is equal to the 30% of the yield strength ($0{,}3\cdot f_y$), otherwise ($h/b \leq 1{,}2$) it is $0{,}5\cdot f_y$.

In the international publications many of different types of geometric imperfections can be found for the modeling of the initial geometry of members. Based on the results of sensitivity analysis it can be stated that one of the most important factors is the geometric imperfection in terms of LT buckling resistance [9]. Therefore, it is an important question that the geometric imperfections with different types and values how can change the carrying capacity of beams. Boissonnade et al. examined this effect in [8], where the LT buckling resistances of beams with end-fork boundary conditions were determined and compared belonging to different initial geometries. We performed tests with similar aim for the case of beams with end-fork boundary conditions and with prevented end-warp. In course of the examination the M-v curves of members given as results of numerical simulations were compared belonging to different initial geometric imperfections, shown on the diagrams in Fig. 2. In Fig. 2-a the M-v curves of the member with HEB300 profile, 8500mm length and S355 material grade can be seen. On the diagram the lines belong to lateral bow imperfection, initial geometry affine to the first eigenmode and two modified first eigenmode with the changing of the ratio between the lateral displacement and rotation (hereinafter $v_0/\varphi_0$), as shown in Fig. 2-a. In all of these cases the total initial lateral displacement of the midpoint of upper flange was specified in value L/1000, where $L$ is the length of the member. With the same test member we carried out numerical simulations to examine the effect of changing the amplitude of the imperfection on the LT buckling resistance, for the case of initial geometry affine to the first eigenmode. The M-v curves belonging to this examination are shown in Fig. 2-b.

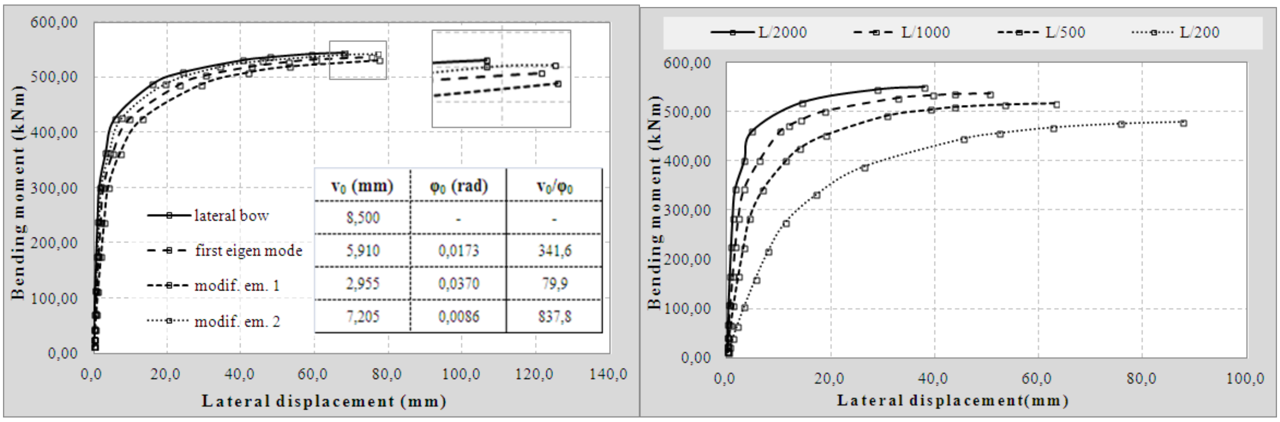



Figure 2. M-v curves of HEB300 with different a- types and b - amplitudes of imperfection

The results of the performed tests show that the LT buckling resistance of the beam with HEB300 (and with also examined IPE500) profile is more sensitive to the rotation type initial imperfection. It can be seen that the decrease of the $v_0/\varphi_0$ ratio of initial geometry induces the decrease of load carrying capacity of the member. Belonging to the different initial geometric imperfections the difference between the maximum values of the bending moment carried by the members is negligible, less than 2,5%. Based on our and international results of numerical tests it can be stated that the type of the geometric imperfection, the initial geometry of the beams has not significant effect on the LT buckling resistance. However, the increasing of the amplitude of imperfection from the value L/2000 to L/200 causes substantial, approximately 13% decrease in the carrying capacity, see in Fig. 2-b. In further numerical tests the initial geometry of the members was defined as affine to the first eigenmode depending on the boundary conditions with the amplitude equal to L/1000, in accordance with the international practice.

With the steel members chosen for the tests geometrical and material non-linear, imperfect (GMNI) analysis were carried out. The behavior and accuracy of the model was verified by the published results of numerical simulations of international research works in [4, 5, 8]. In these papers the authors published LT buckling curves belonging to different profiles, which were compared to the results of our finite element model. Our models of members used for the tests were constructed with the same geometric, material imperfections and material models as these were defined in the international publications. Own and published experimental results were in well approximation, the differences were acceptable.

## 3. Novel method for the LT buckling of simple beams

### *3.1 Ayrton-Perry formula for the theoretical basis of the novel method*

As it was mentioned, the Ayrton-Perry formula provides the basic of the novel method, which was generalized for the case of simple beams. In [1] it was proved that most important stage of the generalization is the properly chosen shape for the initial geometric imperfection. Accordingly, if the first buckling mode is applied for the initial geometry, the generalization of the Ayrton-Perry formula becomes possible - [1]. For the case of the basic model the first buckling mode can be described by the following condition for the imperfection components:

$$\frac{v_0}{\varphi_0} = \frac{M_{cr}}{N_{cr}} \tag{1}$$

where $v_0$ and $\varphi_0$ are the amplitudes of lateral and torsional imperfections with half-sine wave shape, $M_{cr}$ is the elastic critical bending moment of the member and $N_{cr}$ is the elastic critical buckling load about the minor axis.

After having the condition for the initial geometry the first yield criterion can be constructed in terms of the second order internal forces at midspan. Introducing the standard notations for the slenderness $\left[\lambda_{LT} = \sqrt{W_y \cdot f_y / M_{cr}}\right]$ and reduction factor for LT buckling $\left[\chi_{LT} = M_y /(W_y \cdot f_y)\right]$, the standard quadratic form of the Ayrton-Perry formula can be obtained [1]:

$$\chi_{LT}^2 + \chi_{LT} \cdot \left(-1 - \frac{1}{\lambda_{LT}^2} - \frac{1}{\lambda_{LT}^2} \cdot \eta_{LT}\right) + \frac{1}{\lambda_{LT}^2} = 0 \tag{2}$$

where the generalized imperfection factor has the following form:

$$\eta_{LT} = v_0 \cdot \frac{W_y}{W_\omega} + \varphi_0 \cdot \frac{W_y}{W_z} - \varphi_0 \cdot \frac{G \cdot I_t}{M_{cr}} \cdot \frac{W_y}{W_\omega} \tag{3}$$

In the equations $W_y$ , $W_z$ and $W_\omega$ are the elastic major axis, minor axis and warping sectional modules of the cross-section respectively, $f_y$ is the yield strength, $M_y$ is the uniform major axis bending moment, $G$ is the shear modulus of the material and $I_t$ is the St. Venant torsional constant.

It is relevant to state that these derived, theoretical formulae are not appropriate for practical design methods. The formulae define the load-carrying capacity equal to the first yield of the member and do not take into account several effects, e.g. the plastic behavior of the member and the effect of residual stresses. Therefore, a comprehensive work on deterministic and probabilistic calibration is needed to propose an appropriate design method based on the generalized Ayrton-Perry formula.

### *3.2 Novel, Ayrton-Perry based method for the basic model*

For the examination of the utility and efficiency of the generalized Ayrton-Perry formula, a deterministic calibration process was executed for the basic model. The aim of that calibration was to evaluate the applicability of the Ayrton-Perry based formulae. For this evaluation we developed a novel method with calibration for the basic model, and then we examined the possibility of the generalization of this novel method for other boundary condition and other load distribution cases.

To create the buckling database needed to the calibration, an extended numerical test program was carried out, using the finite element model detailed in Section 2, in software ANSYS. For the simulations 100 different members were chosen, which meant 20 different (IPE type, 200, 300, 500, 600 marked, and HEA, HEAA, HEB, HEM types with 300, 450, 600 and 900 mark) profiles and 5 different member lengths for each profiles. The length ($L$) of the beams were determined according to specified, $\lambda_z$ = 0,6; 0,9; 1,2; 1,5 and 2,0 values of slenderness belonging to the buckling about the weak axis. The material grade of the members was S235, with yield strength 235 N/mm$^2$. With the model of the members geometrical and material nonlinear imperfect (GMNI) analysis was carried out.

As the result of the numerical simulations the LT buckling resistance ($M_{b,Rd}$) of the beams was determined which were used to produce the extended database for the members with end-fork boundary conditions subjected to pure bending. From the determined resistances the values of the reduction factor for LT buckling ($\chi_{LT}$) were calculated by Eq. 4:

$$\chi_{LT} = \frac{M_{b,Rd}}{M_{c,Rd}} \tag{4}$$

where $M_{c,Rd}$ is the bending resistance belonging to the cross-sectional carrying capacity. From the $\chi_{LT}$ values using Eq. 5, the formulae of LT buckling curves determined in [1]:

$$\chi_{LT} = \frac{1}{\phi_{LT} + \sqrt{\phi_{LT}^2 - \lambda_{LT}^2}} \qquad \text{where} \qquad \phi_{LT} = 0{,}5 \cdot \left[1 + \eta_{LT} + \lambda_{LT}^2\right] \tag{5}$$

the values of the generalized imperfection factor ($\eta_{LT}$) were calculated. Using the generalized form of $\eta_{LT}$, see in Eq. 3 and the specification for the $v_0/\varphi_0$ ratio, see in Eq. 1, the values of the amplitude of initial, sinusoidal imperfection components were determined. It is important to note, that through the calculations the values of sectional modules of the cross-section were determined belonging to the plastic behavior.

The calculated values of the amplitude of geometrical imperfection were classified based on the cross-section of the members into two groups: group1 means the profiles with h/b ratio over 1,5 and group2 means the other (h/b ≤ 1,5) profiles. For the basic of the calibration the "member length/total lateral displacement" of the midpoint of upper flange (hereinafter: L/v) ratios were chosen. These L/v values were calculated from the results of numerical simulations which are shown on the diagrams in Fig. 3 belonging to the cross-section groups.

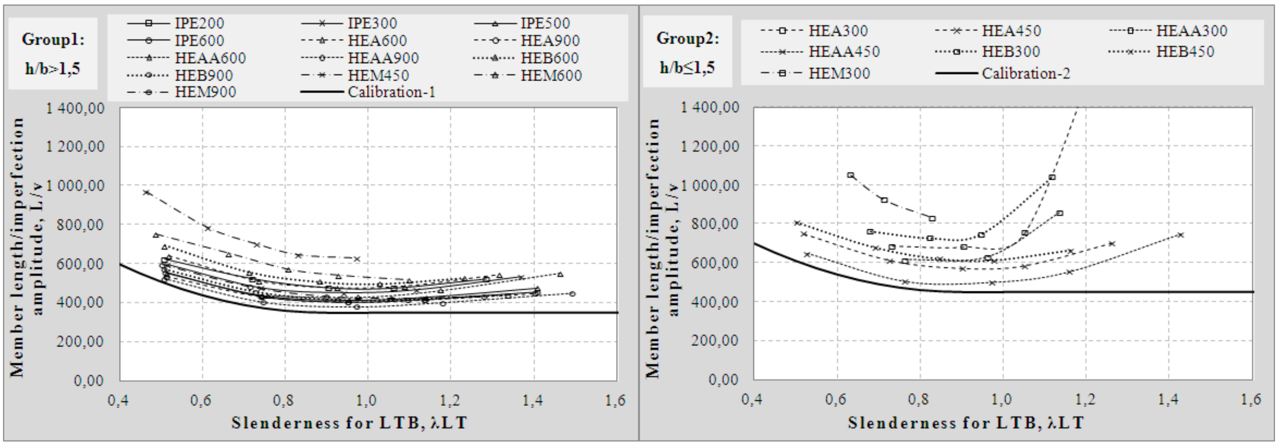



Figure 3. L/v ratios of beams with profiles with height/width ratio a - over 1,5 and b - not over 1,5

On the results shown in Fig. 3 bottom covering curves were fitted belonging to each group. According to the curves, the proposed expression for the calculation of the calibrated, total lateral displacement ($v_{cal}$) of the midpoint of the upper flange is shown in Eq. 6 for the case of profiles with h/b ratio over 1,5 and in Eq. 7 for the case of profiles with h/b ≤ 1,5:

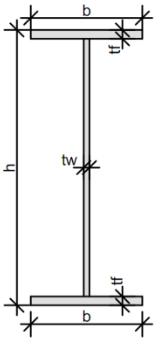
If h/b > 1,5
$$\frac{L}{v_{cal}} = \begin{Bmatrix} 1000 \cdot (\lambda_{LT} - 0{,}9)^2 + 350 \quad if \quad \lambda_{LT} < 0{,}9 \\ 350 \quad if \quad \lambda_{LT} \geq 0{,}9 \end{Bmatrix} \tag{6}$$

If h/b ≤ 1,5
$$\frac{L}{v_{cal}} = \begin{Bmatrix} 1000 \cdot (\lambda_{LT} - 0{,}9)^2 + 450 \quad if \quad \lambda_{LT} < 0{,}9 \\ 450 \quad if \quad \lambda_{LT} \geq 0{,}9 \end{Bmatrix} \tag{7}$$

Using the formulae proposed for the calculation of the total lateral displacement of the midpoint of upper flange and the equations determined in [1], shown by Eq. 2-5 the calibrated value of the LT buckling resistance of different beams can be calculated. We compared the values of reduction factor for LT buckling determined by numerical simulations and calculated with the calibrated formulae. We stated that the differences what can be found to exist between the two types of results are fairly small and are always on the safe side. Therefore, we have a method for the determination of the bending moment carrying capacity of beams with end-fork boundary conditions which is sufficiently accurate and safe.

### *3.3 Example: Ayrton-Perry based method for a member belonging to the basic model (bm)*

The task is the determination of the LT buckling resistance of a sample member with end-fork boundary conditions, loaded by uniform bending. The properties of the cross-section, the member length and the material are given. The resistance has to be determined with numerical simulation, the novel Ayrton-Perry based method and standard method given by the Eurocode, and then these results have to be compared.




| GIVEN PROFILE PROPERTIES | | CALCULATED PROFILE PROPERTIES | | MATERIAL PROPERTIES | |
|---|---|---|---|---|---|
| Profile: IPE500 | | | | Material grade: S235 | |
| - height of the profile: | h = 500mm | A = 113,368cm$^2$ | i = 4,342cm | | $\lambda_1$ = 93,9 |
| - width of the profile: | b = 200mm | $I_z$ = 2137,614cm$^4$ | $W_{pl,y}$ = 2146,153cm$^3$ | - yield strength: | $f_y$ = 235N/mm$^2$ |
| - thickness of the web: | tw = 10,2mm | $I_\omega$ = 1251871,993cm$^6$ | $W_{pl,z}$ = 332,589cm$^3$ | - Young modulus: | E = 210000N/mm$^2$ |
| - thickness of the flange: | tf = 16mm | $I_t$ = 71,734cm$^4$ | $W_{pl,\omega}$ = 8048,65cm$^4$ | - shear modulus: | G = 80770N/mm$^2$ |
| **MEMBER PROPERTIES** | | | | | |
| Slenderness: | $\lambda_z$ = 1,6 | | Member length: | L = $\lambda_z \cdot \lambda_1 \cdot$ i = 652,386cm | |

#### NUMERICAL SIMULATION IN ANSYS

For the numerical simulation the shell finite element model of the given member was defined, and GMNI analysis was carried out. As the result of the test the bending resistance of the member was determined.

Load carrying capacity of the member: $M_{b,Rd,ANSYS} = 255{,}157 kNm$

#### THE AYRTON-PERRY FORMULA BASED METHOD

Using the Ayrton-Perry based formulae determined in [1] and the calibrated equations in Section 3.2, the LT buckling resistance of the member can be calculated. First step is the determination of the slenderness for LT buckling:

Elastic critical normal force: $N_{cr,bm} = \dfrac{\pi^2 \cdot E \cdot I_z}{L^2} = 1040{,}97 kN$

Elastic critical bending moment: $M_{cr,bm} = \dfrac{\pi^2 \cdot E \cdot I_z}{L^2} \cdot \sqrt{\dfrac{I_\omega}{I_z} \cdot \dfrac{L^2 \cdot G \cdot I_t}{\pi^2 \cdot E \cdot I_z}} = 351{,}816 kNm$

Cross-sectional bending resistance: $M_{c,Rk} = W_{pl,y} \cdot f_y = 504{,}346 kNm$

Slenderness for LT buckling: $\lambda_{LT} = \sqrt{\dfrac{M_{c,Rk}}{M_{cr,bm}}}$ $\qquad \lambda_{LT} = 1{,}197$

The h/b ratio of the IPE500 profile is 2,5 therefore, based on the appropriate calibrated equation (Eq.6) the value of the total lateral displacement of the midpoint of upper flange can be calculated:

Calibrated equation: $\dfrac{L}{v_{cal}} = \left| \begin{array}{ll} \left[1000 \cdot (\lambda_{LT} - 0{,}9)^2 + 350\right] & if \quad \lambda_{LT} \le 0{,}9 \\ 350 & if \quad \lambda_{LT} > 0{,}9 \end{array} \right.$ $\qquad v_{cal} = \dfrac{L}{350} = 18{,}64 mm$

From these values, using the condition for the initial shape (Eq.1), the amplitudes of the imperfection components and also the generalized imperfection factor (Eq.3) can be determined:

displacement: $v_{0,cal} = \dfrac{v_{cal}}{1 + \dfrac{N_{cr,bm}}{M_{cr,bm}} \cdot \dfrac{h - t_f}{2}} = 10{,}862 mm$ rotation: $\varphi_{cal} = v_{0,cal} \cdot \dfrac{N_{cr,bm}}{M_{cr,bm}} = 0{,}032$

Generalized imperfection factor: $\eta_{LT,bm} = v_{0,cal} \cdot \dfrac{W_{pl,y}}{W_{pl,\omega}} + \varphi_{0,cal} \cdot \dfrac{W_{pl,y}}{W_{pl,z}} - \varphi_{0,cal} \cdot \dfrac{G \cdot I_t}{M_{cr,bm}} \cdot \dfrac{W_{pl,y}}{W_{pl,\omega}} = 0{,}356$

Applying the formulae of the LT buckling curves determined in [1] (Eq.5) the reduction factor for LT buckling and the LT buckling resistance of the member can be calculated:

$$\phi_{LT,bm} = 0{,}5 \cdot \left(1 + \eta_{LT,bm} + \lambda_{LT}^2\right) = 1{,}395 \qquad \chi_{LT,bm} = \frac{1}{\phi_{LT,bm} + \sqrt{\phi_{LT,bm}^2 - \lambda_{LT}^2}} = 0{,}474$$

LT buckling resistance: $M_{b,Rd,AP} = \chi_{LT,bm} \cdot W_{pl,y} \cdot \dfrac{f_y}{\gamma_{M1}}$ $M_{b,Rd,AP} = 239{,}022 kNm$

## THE EUROCODE METHODS

The LT buckling resistance of the member according to the EN 1993-1-1, section 6.3.2.2. Lateral torsional buckling curves - General case:

imperfection factor curve 'b': $\alpha_{LT} = 0{,}34$ recommended value for curves: $\lambda_{LT,0} = 0{,}2$

$$\phi_{LT} = 0{,}5 \cdot \left(1 + \alpha_{LT} \cdot \left(\lambda_{LT} - \lambda_{LT,0}\right) + \lambda_{LT}^2\right) = 1{,}386 \qquad \chi_{LT,bm} = \frac{1}{\phi_{LT,bm} + \sqrt{\phi_{LT,bm}^2 - \lambda_{LT}^2}} = 0{,}48$$

LT buckling resistance: $M_{b,Rd,EC} = \chi_{LT,bm} \cdot W_{pl,y} \cdot \dfrac{f_y}{\gamma_{M1}}$ $M_{b,Rd,EC} = 241{,}878 kNm$

The LT buckling resistance of the member according to the EN 1993-1-1, section 6.3.2.3. Lateral torsional buckling curves for rolled sections or equivalent welded sections:

imperfection factor curve 'c': $\alpha_{LT} = 0{,}49$ recommended values for curves: $\lambda_{LT,0} = 0{,}4$ $\beta = 0{,}75$

$$\phi_{LT} = 0{,}5 \cdot \left(1 + \alpha_{LT} \cdot \left(\lambda_{LT} - \lambda_{LT,0}\right) + \beta \cdot \lambda_{LT}^2\right) = 1{,}233 \qquad \chi_{LT,bm} = \frac{1}{\phi_{LT,bm} + \sqrt{\phi_{LT,bm}^2 - \beta \cdot \lambda_{LT}^2}} = 0{,}526$$

LT buckling resistance: $M_{b,Rd,EC} = \chi_{LT,bm} \cdot W_{pl,y} \cdot \dfrac{f_y}{\gamma_{M1}}$ $M_{b,Rd,EC} = 265{,}286 kNm$

## COMPARISON OF THE RESULTS

For the evaluation of the novel method the LTB resistance of the tested member was determined by the EN1993-1-1 methods too. The results of the Eurocode and the novel, Ayrton-Perry formula based method are compared with the results of the numerical simulation. The results are summarized in Table 1.

Table 1**:** Comparison of LTB resistances of the basic model sample

| Determination method | LTB resistance (kNm) | Difference on safe side (%) |
|---|---|---|
| ANSYS simulation | 255,157 | (basic) |
| APF based method | 239,022 | 6,750 |
| EC - 6.3.2.2 | 241,878 | 5,490 |
| EC - 6.3.2.3 | 265,286 | -3,818 |

# 4. Generalization of the novel method for beams with prevented end-warp

## *4.1 Generalization based on the theory of elastic extrapolation*

In Section 3.3 it was demonstrated how the novel, Ayrton-Perry formula based method works for the basic model. Hereinafter, it is examined how can be this novel method generalized for the case of members with end-fork but end-warp preventing boundary conditions, loaded by uniform bending. The basic idea of the generalization was the theory of elastic extrapolation. The theory says that if the buckling behavior belonging to the ideal models of two steel members is similar, the real carrying capacity of the two members is related. In terms of the method this means that first the slenderness of the examined, end-warp prevented member has to be determined. Then, with this slenderness an equivalent, basic model member has to be defined which the demonstrated method has to be carried out with. According to the theory of the elastic extrapolation the LT buckling resistance, calculated for this equivalent member has to be a good approximation for the originally examined member with prevented end-warp.

## *4.2 Example: Ayrton-Perry based method for an end-warp prevented member*

The task is the determination of the LT buckling resistance of a sample member with end-fork but end-warp preventing boundary conditions, loaded by uniform bending. The properties of the cross-section, the member length and the material are the same as given in Section 3.3. The resistance has to be determined with numerical simulation, the novel Ayrton-Perry based method and standard method given by the Eurocode, and then these results have to be compared.

### NUMERICAL SIMULATION IN ANSYS

For the numerical simulation the shell finite element model of the given member was defined, and GMNI analysis was carried out. As the result of the test the bending resistance of the member was determined.

Load carrying capacity of the member: $M_{b,Rd,ANSYS} = 347{,}292 kNm$

### THE AYRTON-PERRY FORMULA BASED METHOD

Using the Ayrton-Perry based formulae determined in [1] and the calibrated equations in Section 3.2, the LT buckling resistance of the member can be calculated. First step is the determination of the slenderness for LT buckling:

Boundary conditions: - prevention for end rotation: $k = 0{,}85$

- prevention for end-warping: $k_w = 0{,}5$

Elastic critical bending moment: $$M_{cr,bm} = \frac{\pi^2 \cdot E \cdot I_z}{(k \cdot L)^2} \cdot \sqrt{\left(\frac{k}{k_w}\right)^2 \frac{I_\omega}{I_z} \cdot \frac{(k \cdot L)^2 \cdot G \cdot I_t}{\pi^2 \cdot E \cdot I_z}} = 659{,}409 kNm$$

Cross-sectional bending resistance: $M_{c,Rk} = W_{pl,y} \cdot f_y = 504{,}346 kNm$

Slenderness for LT buckling: $$\lambda_{LT} = \sqrt{\frac{M_{c,Rk}}{M_{cr,bm}}} \qquad \lambda_{LT} = 0{,}875$$

According to the theory of elastic extrapolation the equivalent, basic model member has to be defined. The member length of the equivalent beam:

Elastic critical bending moment for the basic model: $$M_{cr,bm} = \frac{M_{c,Rd}}{\lambda_{LT}^2} = 659{,}409 kNm$$

Equivalent member length: $L_{bm} = \sqrt{0{,}5 \cdot \left[ \frac{\pi^2 \cdot E \cdot I_z \cdot G \cdot I_t}{{M_{cr,bm}}^2} \cdot \sqrt{\frac{\pi^4 \cdot E^2 \cdot I_z^2}{{M_{cr,bm}}^2} \cdot \left( \frac{G^2 \cdot I_t^2}{{M_{cr,bm}}^2} + 4 \cdot \frac{I_\omega}{I_z} \right)} \right]} = 441{,}329 cm$

Elastic critical normal force: $N_{cr,bm} = \frac{\pi^2 \cdot E \cdot I_z}{L_{bm}^2} = 2274{,}699 kN$

The h/b ratio of the IPE500 profile is 2,5 therefore, based on the appropriate calibrated equation (Eq.6) the value of the total lateral displacement of the midpoint of upper flange can be calculated:

Calibrated equation: $\frac{L}{v_{cal}} = \left| \begin{array}{ll} \left[1000 \cdot (\lambda_{LT} - 0{,}9)^2 + 350\right] & if \quad \lambda_{LT} \le 0{,}9 \\ 350 & if \quad \lambda_{LT} > 0{,}9 \end{array} \right.$

Total lateral displacement of the upper flange: $v_{cal} = \frac{L}{1000 \cdot (\lambda_{LT} - 0{,}9)^2 + 350} = 12{,}586 mm$

From these values, using the condition for the initial shape (Eq.1), the amplitudes of the imperfection components and also the generalized imperfection factor (Eq.3) can be determined:

displacement: $v_{0,cal} = \frac{v_{cal}}{1 + \frac{N_{cr,bm}}{M_{cr,bm}} \cdot \frac{h - t_f}{2}} = 6{,}86 mm$ rotation: $\varphi_{cal} = v_{0,cal} \cdot \frac{N_{cr,bm}}{M_{cr,bm}} = 0{,}024$

Generalized imperfection factor: $\eta_{LT,bm} = v_{0,cal} \cdot \frac{W_{pl,y}}{W_{pl,\omega}} + \varphi_{0,cal} \cdot \frac{W_{pl,y}}{W_{pl,z}} - \varphi_{0,cal} \cdot \frac{G \cdot I_t}{M_{cr,bm}} \cdot \frac{W_{pl,y}}{W_{pl,\omega}} = 0{,}28$

Applying the formulae of the LT buckling curves determined in [1] (Eq.5) the reduction factor for LT buckling and the LT buckling resistance of the member can be calculated:

$$\phi_{LT,bm} = 0{,}5 \cdot \left(1 + \eta_{LT,bm} + \lambda_{LT}^2\right) = 1{,}023 \qquad \chi_{LT,bm} = \frac{1}{\phi_{LT,bm} + \sqrt{\phi_{LT,bm}^2 - \lambda_{LT}^2}} = 0{,}644$$

LT buckling resistance: $M_{b,Rd,AP} = \chi_{LT,bm} \cdot W_{pl,y} \cdot \frac{f_y}{\gamma_{M1}}$ $M_{b,Rd,AP} = 324{,}905 kNm$

## THE EUROCODE METHODS

The LT buckling resistance of the member according to the EN 1993-1-1, section 6.3.2.2. Lateral torsional buckling curves - General case:

imperfection factor curve 'b': $\alpha_{LT} = 0{,}34$ recommended value for curves: $\lambda_{LT,0} = 0{,}2$

$$\phi_{LT} = 0{,}5 \cdot \left(1 + \alpha_{LT} \cdot (\lambda_{LT} - \lambda_{LT,0}) + \lambda_{LT}^2\right) = 0{,}997 \qquad \chi_{LT,bm} = \frac{1}{\phi_{LT,bm} + \sqrt{\phi_{LT,bm}^2 - \lambda_{LT}^2}} = 0{,}678$$

LT buckling resistance: $M_{b,Rd,EC} = \chi_{LT,bm} \cdot W_{pl,y} \cdot \frac{f_y}{\gamma_{M1}}$ $M_{b,Rd,EC} = 341{,}696 kNm$

The LT buckling resistance of the member according to the EN 1993-1-1, section 6.3.2.3. Lateral torsional buckling curves for rolled sections or equivalent welded sections:

imperfection factor curve 'c': $\alpha_{LT} = 0{,}49$ recommended values for curves: $\lambda_{LT,0} = 0{,}4$ $\beta = 0{,}75$

$$\phi_{LT} = 0{,}5 \cdot \left(1 + \alpha_{LT} \cdot (\lambda_{LT} - \lambda_{LT,0}) + \beta \cdot \lambda_{LT}^2\right) = 0{,}903 \qquad \chi_{LT,bm} = \frac{1}{\phi_{LT,bm} + \sqrt{\phi_{LT,bm}^2 - \beta \cdot \lambda_{LT}^2}} = 0{,}717$$

LT buckling resistance: $M_{b,Rd,EC} = \chi_{LT,bm} \cdot W_{pl,y} \cdot \frac{f_y}{\gamma_{M1}}$ $M_{b,Rd,EC} = 361{,}616 kNm$

**COMPARISON OF THE RESULTS**

For the evaluation of the novel method the LTB resistance of the tested member was determined by the EN1993-1-1 methods too. The results of the Eurocode and the novel, APF based method are compared with the results of the numerical simulation. The results are summarized in Table 2.

Table 2**:** Comparison of LTB resistances of the prevented end-warp sample

| Determination method | LTB resistance (kNm) | Difference on safe side (%) |
|---|---|---|
| ANSYS simulation | 347,292 | (basic) |
| APF based method | 324,905 | 6,890 |
| EC - 6.3.2.2 | 341,696 | 1,638 |
| EC - 6.3.2.3 | 361,616 | -3,961 |

## 5. Generalization of the novel method for simple beams loaded by triangular moment distribution

### *5.1 Generalization based on the theory of elastic extrapolation*

The generalization of the novel method for the case of members with end-fork boundary conditions and loaded by triangular moment distribution was based on the theory of elastic extrapolation, similar to the Section 4. However, in this case the methodology is more complicated. Due to the triangular moment distribution the critical cross-section is not known, it shifts from the middle of the beam towards the maximum value of the bending moment. Therefore, the member has to be divided into segments by the appropriate number of sections and each chosen cross-section has to be examined. For each cross-section the non dimensional slenderness can be determined by Eq.8:

$$\lambda_i = \sqrt{\frac{\alpha_{ult,i}}{\alpha_{cr}}} \tag{8}$$

where $\lambda_i$ is the slenderness of the i-th cross-section, $\alpha_{ult,i}$ is the minimum load amplifier of the cross-sectional load to reach the characteristic resistance of the cross-section and $\alpha_{cr}$ is the minimum amplifier of the load of the beam to reach the elastic critical resistance of the member. Similar to the Section 4, with the determined slenderness an equivalent, basic model member has to be defined for each cross-section. Then, with these basic model members a generalized form of the novel method has to be carried out.

The generalized form of the novel method means, that first a load value ($M_{Ed}$) is supposed on the member. For this load value a load amplification factor ($\alpha_{b,Rd,i}$) has to be determined for each cross-section. This amplification factor takes into account two effects: the load distribution and the first buckling shape of the member. According to the methodology, the $\alpha_{b,Rd,i}$ amplification factor for each cross-section has to be determined as the multiplication of two factors: the $\alpha_{ult,i}$ load amplifier and the $\chi_{LT,i}$ reduction factor for LT buckling. The $\chi_{LT,i}$ reduction factor has to be calculated for the equivalent, basic model member which is defined belonging to the i-th cross-section. The calculation method is similar to the method detailed in the example in Section 4.2. The only difference is that the value of the generalized imperfection factor has to be weighted according to the first buckling mode of the member. Multiplying the calculated values of $\alpha_{ult,i}$ load amplifier and the $\chi_{LT,i}$ reduction factor the values of $\alpha_{b,Rd,i}$ amplification factor can be determined for each cross-section. The cross-section belonging to the minimum value of the amplification factors can be defined as the critical. Multiplying the minimum value of the amplification factors ($\alpha_{b,Rd,min}$) with the applied load value ($M_{Ed}$) the LT buckling resistance of the member can be calculated. An example for this methodology is detailed in Section 5.2.

### *5.2 Example: Ayrton-Perry based method for triangular moment distribution*

The task is the determination of the LT buckling resistance of a sample member with end-fork boundary conditions, loaded by triangular moment distribution. The properties of the cross-section, the member length and the material are the same as given in Section 3.3. The resistance has to be determined with numerical simulation, the novel Ayrton-Perry based method and standard methods given by the Eurocode, and then these results have to be compared.

#### NUMERICAL SIMULATION IN ANSYS

For the numerical simulation the shell finite element model of the given member was defined, and GMNI analysis was carried out. As the results of the test the bending resistance and the first buckling mode of the member were determined.

Load carrying capacity of the member: $M_{b,Rd,ANSYS} = 423{,}275 kNm$

#### THE AYRTON-PERRY FORMULA BASED METHOD

The value of the bending moment on one of the ends of the member is supposed to be 100 kNm. The amplification factor belonging to the LT buckling resistance has to be determined. For the examination the member is divided into 20 segments. The amplification factor has to be determined for every cross-section, depending on the load distribution and the first buckling shape. The minimum value of the factors will be used for the calculation of the LT buckling resistance.

The effect of the cross-sectional bending load is taken into account by the minimum load amplifier:

Bending moment on one of the ends of the beam: $M_{Ed} = 100 kNm$

Cross-sectional bending resistance: $M_{c,Rk} = W_{pl,y} \cdot f_y = 504{,}346 kNm$

Load amplifier for the cross-sectional, characteristic resistance: $\alpha_{ult,i} = \dfrac{M_{c,Rk}}{M_{Ed,i}}$

The reduction factor for LT buckling is calculated for each cross-section. For the calculation, the values of slenderness belonging to each cross-section are determined and the equivalent, basic model members are defined.

Elastic critical bending moment: $M_{cr} = 1{,}879 \cdot \dfrac{\pi^2 \cdot E \cdot I_z}{L^2} \cdot \sqrt{\dfrac{I_\omega}{I_z} \cdot \dfrac{L^2 \cdot G \cdot I_t}{\pi^2 \cdot E \cdot I_z}} = 661{,}062 kNm$

Load amplifier for the elastic critical beam resistance: $\alpha_{cr} = \dfrac{M_{cr}}{M_{Ed}}$ $\qquad \alpha_{cr} = 6{,}611$

Slenderness for LT buckling: $\lambda_{LT,i} = \sqrt{\dfrac{\alpha_{ult,i}}{\alpha_{cr}}}$

According to the theory of elastic extrapolation the equivalent, basic model member has to be defined belonging to each cross-sections. The member length of the equivalent beams:

Elastic critical bending moment for the basic model: $M_{cr,bm,i} = \dfrac{M_{c,Rd}}{\lambda_{LT,i}^2}$

Equivalent member length: $$L_{bm,i} = \sqrt{0{,}5 \cdot \left[ \frac{\pi^2 \cdot E \cdot I_z \cdot G \cdot I_t}{M_{cr,bm}{}^2} \cdot \sqrt{\frac{\pi^4 \cdot E^2 \cdot I_z^2}{M_{cr,bm}{}^2} \cdot \left( \frac{G^2 \cdot I_t^2}{M_{cr,bm}{}^2} + 4 \cdot \frac{I_\omega}{I_z} \right)} \right]}$$

Elastic critical normal force: $$N_{cr,bm,i} = \frac{\pi^2 \cdot E \cdot I_z}{L_{bm,i}^2}$$

The h/b ratio of the IPE500 profile is 2,5 therefore, based on the appropriate calibrated equation (Eq.6) the values of the total lateral displacement of the midpoint of upper flange can be calculated:

Calibrated equation: $$\frac{L}{v_{cal}} = \left| \begin{array}{ll} \left[1000 \cdot (\lambda_{LT} - 0{,}9)^2 + 350\right] & if \quad \lambda_{LT} \le 0{,}9 \\ 350 & if \quad \lambda_{LT} > 0{,}9 \end{array} \right.$$

Total lateral displacement of the upper flange: $$v_{cal,i} = \frac{L_{bm,i}}{\left( \dfrac{L_{bm,i}}{v_{cal,i}} \right)}$$

From these values, using the condition for the initial shape (Eq.1), the amplitudes of the imperfection components and also the generalized imperfection factor (Eq.3) can be determined for the virtual, equivalent, basic model members belonging to each cross-sections:

displacement: $$v_{0,cal,i} = \frac{v_{cal,i}}{1 + \dfrac{N_{cr,bm,i}}{M_{cr,bm,i}} \cdot \dfrac{h - t_f}{2}}$$

rotation: $$\varphi_{cal,i} = v_{0,cal,i} \cdot \frac{N_{cr,bm,i}}{M_{cr,bm,i}}$$

Generalized imperfection factor: $$\eta_{LT,bm,i} = v_{0,cal,i} \cdot \frac{W_{pl,y}}{W_{pl,\omega}} + \varphi_{0,cal,i} \cdot \frac{W_{pl,y}}{W_{pl,z}} - \varphi_{0,cal,i} \cdot \frac{G \cdot I_t}{M_{cr,bm,i}} \cdot \frac{W_{pl,y}}{W_{pl,\omega}}$$

The values of the generalized imperfection factor have to be weighted according to the first buckling mode of the member:

lateral displacement of the cross-section: $v_i$ maximum displacement: $v_{\max}$

The weighted values of the generalized imperfection factor: $$\eta_{LT,i} = \eta_{LT,bm,i} \cdot \frac{v_i}{v_{\max}}$$

Applying the formulae of the LT buckling curves determined in [1] (Eq.5) the reduction factors for LT buckling can be calculated:

$$\phi_{LT,i} = 0{,}5 \cdot \left(1 + \eta_{LT,i} + \lambda_{LT}^2\right) \qquad \chi_{LT,i} = \frac{1}{\phi_{LT,i} + \sqrt{\phi_{LT,i}^2 - \lambda_{LT,i}^2}}$$

After calculating the values of the minimum load amplifier and reduction factors, multiplying these values the amplification factors can be determined for each cross-section:

$$\alpha_{b,Rd,i} = \alpha_{ult,i} \cdot \chi_i$$

Table 3**:** The results of the calculation

| Cross-section number | $M_{Ed,i}$ (kNm) | $\alpha_{ult}$ | $\alpha_{cr}$ | $\lambda_{LT}$ | $M_{cr,bm}$ | $L_{bm}$ | $L/v_{cal}$ | $v_{cal}$ | $v_{0,cal}$ | $\eta_{LT,bm}$ | $\eta_{LT}$ | $\Phi_{LT}$ | $\chi_{LT}$ | $\alpha_{b,Rd}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 100,00 | 5,043 | | 0,873 | 661,062 | 4406,780 | 350,704 | 12,566 | 7,322 | 0,240 | 0,000 | 0,881 | 1,000 | 5,043 |
| 1 | 95,00 | 5,309 | | 0,896 | 628,009 | 4542,494 | 350,015 | 12,978 | 7,563 | 0,248 | 0,046 | 0,924 | 0,868 | 4,610 |
| 2 | 90,00 | 5,604 | | 0,921 | 594,956 | 4691,282 | 350,000 | 13,404 | 7,811 | 0,256 | 0,093 | 0,970 | 0,783 | 4,390 |
| 3 | 85,00 | 5,933 | | 0,947 | 561,903 | 4855,344 | 350,000 | 13,872 | 8,084 | 0,265 | 0,139 | 1,018 | 0,719 | 4,265 |
| 4 | 80,00 | 6,304 | | 0,977 | 528,850 | 5037,424 | 350,000 | 14,393 | 8,387 | 0,275 | 0,184 | 1,069 | 0,665 | 4,195 |
| 5 | 75,00 | 6,725 | | 1,009 | 495,797 | 5240,993 | 350,000 | 14,974 | 8,726 | 0,286 | 0,226 | 1,122 | 0,620 | 4,170 |
| 6 | 70,00 | 7,205 | | 1,044 | 462,744 | 5470,516 | 350,000 | 15,630 | 9,108 | 0,298 | 0,264 | 1,177 | 0,581 | 4,188 |
| 7 | 65,00 | 7,759 | | 1,083 | 429,691 | 5731,836 | 350,000 | 16,377 | 9,543 | 0,313 | 0,298 | 1,236 | 0,546 | 4,240 |
| 8 | 60,00 | 8,406 | | 1,128 | 396,637 | 6032,764 | 350,000 | 17,236 | 10,044 | 0,329 | 0,326 | 1,299 | 0,515 | 4,326 |
| 9 | 55,00 | 9,170 | | 1,178 | 363,584 | 6383,992 | 350,000 | 18,240 | 10,629 | 0,348 | 0,348 | 1,368 | 0,485 | 4,445 |
| 10 | 50,00 | 10,087 | 6,611 | 1,235 | 330,531 | 6800,576 | 350,000 | 19,430 | 11,323 | 0,371 | 0,366 | 1,446 | 0,455 | 4,591 |
| 11 | 45,00 | 11,208 | | 1,302 | 297,478 | 7304,431 | 350,000 | 20,870 | 12,162 | 0,398 | 0,378 | 1,537 | 0,425 | 4,762 |
| 12 | 40,00 | 12,609 | | 1,381 | 264,425 | 7928,766 | 350,000 | 22,654 | 13,201 | 0,433 | 0,385 | 1,646 | 0,393 | 4,961 |
| 13 | 35,00 | 14,410 | | 1,476 | 231,372 | 8726,429 | 350,000 | 24,933 | 14,529 | 0,476 | 0,387 | 1,783 | 0,359 | 5,177 |
| 14 | 30,00 | 16,812 | | 1,595 | 198,319 | 9786,853 | 350,000 | 27,962 | 16,295 | 0,534 | 0,384 | 1,963 | 0,322 | 5,408 |
| 15 | 25,00 | 20,174 | | 1,747 | 165,266 | 11273,847 | 350,000 | 32,211 | 18,771 | 0,615 | 0,378 | 2,215 | 0,280 | 5,640 |
| 16 | 20,00 | 25,217 | | 1,953 | 132,212 | 13521,197 | 350,000 | 38,632 | 22,512 | 0,738 | 0,371 | 2,593 | 0,233 | 5,868 |
| 17 | 15,00 | 33,623 | | 2,255 | 99,159 | 17321,546 | 350,000 | 49,490 | 28,840 | 0,945 | 0,361 | 3,224 | 0,181 | 6,083 |
| 18 | 10,00 | 50,435 | | 2,762 | 66,106 | 25084,134 | 350,000 | 71,669 | 41,764 | 1,368 | 0,352 | 4,491 | 0,125 | 6,280 |
| 19 | 5,00 | 100,869 | | 3,906 | 33,053 | 48924,314 | 350,000 | 139,784 | 81,457 | 2,669 | 0,346 | 8,303 | 0,064 | 6,454 |
| 20 | 0,00 | | | | | | | | | | | | | |

Choosing the minimum value of the amplification factor the LT buckling resistance of the member can be calculated. In the sample:

The minimum value of the amplification factor: $\alpha_{b,Rd,\min} = 4{,}17027$

LT buckling resistance: $M_{b,Rd,AP} = \alpha_{b,Rd,\min} \cdot M_{Ed} = 417{,}027 kNm$

## THE EUROCODE METHODS

The LT buckling resistance of the according to the EN 1993-1-1, section 6.3.2.2. Lateral torsional buckling curves - General case:

Slenderness for LT buckling: $\lambda_{LT} = \sqrt{\dfrac{M_{c,Rk}}{M_{cr}}}$ $\qquad \lambda_{LT} = 0{,}873$

imperfection factor curve 'b': $\alpha_{LT} = 0{,}34$ recommended value for curves: $\lambda_{LT,0} = 0{,}2$

$$\phi_{LT} = 0{,}5 \cdot \left(1 + \alpha_{LT} \cdot \left(\lambda_{LT} - \lambda_{LT,0}\right) + \lambda_{LT}^2\right) = 0{,}996 \qquad \chi_{LT,bm} = \frac{1}{\phi_{LT,bm} + \sqrt{\phi_{LT,bm}^2 - \lambda_{LT}^2}} = 0{,}678$$

LT buckling resistance: $M_{b,Rd,EC} = \chi_{LT} \cdot W_{pl,y} \cdot \dfrac{f_y}{\gamma_{M1}}$ $\qquad M_{b,Rd,EC} = 342{,}049 kNm$

The LT buckling resistance of the member according to the EN 1993-1-1, section 6.3.4. General method for lateral and lateral torsional buckling of structural components:

Load amplifiers: $\alpha_{ult,k} = \dfrac{M_{c,Rk}}{M_{Ed}} = 5{,}043$ $\qquad \alpha_{cr,op} = \dfrac{M_{cr}}{M_{Ed}} = 6{,}611$

Global non dimensional slenderness: $\lambda_{op} = \sqrt{\dfrac{\alpha_{ult,k}}{\alpha_{cr,op}}}$ $\qquad \lambda_{op} = 0{,}873$

Reduction factor according to 6.3.2.3:

imperfection factor curve 'c': $\alpha_{LT} = 0{,}49$ recommended values for curves: $\lambda_{LT,0} = 0{,}4$ $\beta = 0{,}75$

$$\phi_{LT} = 0{,}5 \cdot \left(1 + \alpha_{LT} \cdot \left(\lambda_{LT} - \lambda_{LT,0}\right) + \beta \cdot \lambda_{LT}^2\right) = 0{,}902 \qquad \chi_{LT} = \frac{1}{\phi_{LT,bm} + \sqrt{\phi_{LT,bm}^2 - \beta \cdot \lambda_{LT}^2}} = 0{,}718$$

Correction factor depending on moment distribution: $k_c = 1/1{,}33 = 0{,}752$

Modification factor: $f = 1 - 0{,}5 \cdot (1 - k_c) \cdot \left[1 - 2 \cdot (\lambda_{LT} - 0{,}8)^2\right] = 0{,}877$

Modified reduction factor: $\chi_{LT,\mathrm{mod}} = \dfrac{\chi_{LT,bm}}{f} = 0{,}818$

LT buckling resistance: $M_{b,Rd,EC} = \chi_{LT,\mathrm{mod}} \cdot W_{pl,y} \cdot \dfrac{f_y}{\gamma_{M1}}$ $M_{b,Rd,EC} = 412{,}526 kNm$

**COMPARISON OF THE RESULTS**

For the evaluation of the novel method the LTB resistance of the tested member was determined by the Eurocode methods too. The results of the Eurocode and the novel, APF based method are compared with the results of the numerical simulation. The results are summarized in Table 4.

Table 4**:** Comparison of LTB resistances of the triangular moment distribution sample

| Determination method | LTB resistance (kNm) | Difference on safe side (%) |
|---|---|---|
| ANSYS simulation | 423,275 | (basic) |
| APF based method | 417,027 | 1,498 |
| EC - 6.3.2.2 | 342,049 | 23,747 |
| EC - 6.3.4 | 412,526 | 2,605 |

## 6. Conclusions

In this paper the finite element model of steel members used for the numerical tests is detailed. We examined the effect of changing the types and amplitudes of material imperfections and geometrical imperfections. It was stated that the distribution types of residual stresses and shapes of initial geometry of the beams have not significant effect on the LT buckling resistance.

In the present paper we described an extended and improved calibration process for the Ayrton-Perry formula derived for the basic model in [1]. We proposed equations for the calculation of the total lateral displacement of the midpoint of upper flange. With the new proposals and the formulae based on the Ayrton-Perry formula a sufficiently safe and accurate method is given for the examination of the LT buckling resistance of bended beams with end-fork boundary conditions.

Based on the theory of elastic extrapolation the generalization of the novel method was introduced for beams with prevented end-warp and also for beams with triangular moment distribution. The application of the method was demonstrated through examples. Therefore, it is stated that the novel, Ayrton-Perry formula based method and the LT buckling curves calibrated for the basic model are also appropriate for the case of beams with prevented end-warp and triangular moment distribution to calculate the bending moment carrying capacity. Naturally, more extensive numerical test program and calibration is needed to examine the validity of the theory of elastic extrapolation for the case of beams with other boundary conditions and load cases. Comparing the results of the novel method with the results of numerical simulations and standard methods it can be seen that the APF based resistances are properly accurate. Examined the existing methods it can be stated that the novel, APF based method is appropriate for LTB type design problems and beneficial for practical application.

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