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Vertical Vibrations of 3D Structure Caused by Moving Load

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Abstract

This paper presents a numerical study of the dynamic response of 3D structure subjected to a vertical moving load using substructure approach in the frequency domain. The dynamic system consists of two substructures, the unbounded soil region and the 3D structure. The soil is considered as an elastic, homogeneous and isotropic half-space. The impedance functions of the soil, as well as the dynamic response of the half-space to a moving vehicle are determined using the Integral Transform Method (ITM). The vertical vibrations of the structure are calculated using the Spectral Element Method. Based on this theoretical consideration, the numerical analysis is carried out and the obtained results are presented.

Keywords: soil-structure interaction, frequency domain, dynamic response, spectral element, integral transform, moving load, frame structure.

1. Introduction

Modern era brought a great development in the field of urban planning and transportation systems. Needs for easily accessible, available and mobile public transport have produced higher level of traffic induced vibrations in buildings. Nowadays, these vibrations are mainly considered as a nuisance in residential buildings, although they can produce minor damages of the monumental historical buildings and sensitive equipment malfunction in the industrial buildings. Hence, the influence of traffic induced vibrations on buildings and humans has become object of interest of environmental problem researchers and traffic system designers in the development countries.
The rail/road - vehicle system imperfections, such as road and wheel roughness, are the main sources of traffic induced ground vibrations. These vibrations induce waves that propagate through the soil and affect the surrounding buildings. Prediction of traffic induced vibrations in buildings is complex and requires the following issues to be addressed properly: emission (excitation by traffic), transmission (wave propagation through the soil), and immission (transfer of excitation into a building).

In the last decade, comprehensive investigations have been carried out to develop a numerical model for reliable prediction of traffic induced building vibrations. Most of them are based on the substructure approach, where the soil and the structure are analyzed separately. Different methods of analysis may be applied for evaluation of transient or steady-state response of the soil induced by moving sources. For simple geometry of the soil region analytical or semi-analytical solutions in the wavenumber–frequency domain are the most applicable, like Integral Transform Method (ITM),

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[1]. As an alternative to the semi-analytical solution, the thin-layer method can be employed, [2]. For the analysis of subsoil with complex geometry numerical methods, like Finite Element Method (FEM), [3], or Boundary Element Method (BEM), [4], are available in the time or frequency domain. In the FEM the infinite soil region can be modeled using different types of transmitting boundaries, [5]. The overview of the numerical methods for the analysis of ground vibrations due to moving load is given by Andersen, [6].

The structure is usually modeled using the FEM, in time or frequency domain. The accuracy of the results depends on the size of finite elements, which should be less than λ/8, where λ is the wave length. This might cause an increase in the number of finite elements and therefore more computational efforts. As an alternative to the FEM, the Spectral Element Method (SEM) can be used to analyze vibration problems in buildings [7]. The interpolation functions of a spectral element are the exact solutions of the governing equation of motion, so, only one element can exactly represent the dynamic behavior of a structural member at any frequency. Thus, the application of spectral elements significantly reduces the number of the unknowns, increases the accuracy of the numerical results and decreases the computational time, [8]. In addition, the frequency dependent dynamic stiffness of the soil-foundation system can be easily incorporated in the spectral element model [9]. Despite of many advantages, spectral elements are still not sufficiently used in the analysis of vibrations.

In this paper presented is the analysis of the vertical response of simple 3D structure due to the unit moving force on the half-space surface using spectral element method. The structure is one storey building, consisted of a plate supported by four columns. The soil is considered as an elastic, homogeneous and isotropic half-space. The solution of the wave propagation in the half-space is based on the ITM, while the dynamic response of the structure is obtained using the SEM. The main objectives of the paper are: (1) to calculate wave propagation in elastic half-space using the ITM; (2) to apply the ITM to calculate the dynamic stiffness of the soil-structure interface; (3) to calculate the vertical response of the soil-structure system due to moving force, using the SEM for structural modeling.

2. Half-space solution according to ITM

The ITM was applied in the last years to several problems of half-space dynamics [1], and soil-structure interaction [10], in particular, to problems of train/track/soil interaction, [12],[13]. Method is based on the Helmholz's decomposition of the Lamé's equations and their threefold Fourier transform from the time-space to the frequency wavenumber domain. Therefore it is restricted to linear systems and to the frequency domain analysis. The short description of this method will be presented in the following.

The Lamé's equations of motion of the continuum

$$\mu\nabla^2\mathbf{u}+(\lambda+\mu)\nabla\nabla\cdot\mathbf{u}=\rho\ddot{\mathbf{u}} \tag{1}$$

can be brought into the form of wave equations

$$\nabla^2\varphi=\frac{1}{c_p^2}\ddot{\varphi},\qquad \nabla^2\boldsymbol{\psi}=\frac{1}{c_s^2}\ddot{\boldsymbol{\psi}} \tag{2}$$

if the displacement vector is expressed by the scalar field φ and the vector field $\boldsymbol{\psi}$, according to Helmholtz's principle, as

$$\nabla u=\nabla\varphi+\nabla\times\boldsymbol{\psi}. \tag{3}$$

In Eqs. (2) c_p and c_s are the velocities of the dilatational and shear waves, respectively

$$c_p^2 = \frac{\lambda + 2\mu}{\rho}, \qquad c_s^2 = \frac{\mu}{\rho} \tag{4}$$

where ρ is the mass density of the material and λ and μ are the Lamé's constants.

If we assume that $\psi_z = 0$, displacement components can be obtained form Eq. (3) in the following form

$$\begin{aligned} u_x &= \varphi_{,x} - \psi_{y,z} \\ u_x &= \varphi_{,y} - \psi_{x,z} \\ u_z &= \varphi_{,z} - \psi_{x,y} + \psi_{y,x} \end{aligned} \quad . \tag{5}$$

By a threefold Fourier transform

$$\hat{f}(k_x, k_y, \omega) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(x,y,t) e^{-i(k_x x + k_y y + \omega t)} dxdydt \,, \tag{6}$$

Eqs. (2) can be transformed into a system of 3 decoupled ordinary differential equations in frequency wavenumber domain

$$\begin{aligned} -(k_x^2 + k_y^2 - k_p^2)\hat{\varphi} + \frac{\partial^2 \hat{\varphi}}{\partial z^2} &= 0 \\ -(k_x^2 + k_y^2 - k_s^2)\hat{\psi}_i + \frac{\partial^2 \hat{\psi}_i}{\partial z^2} &= 0 \end{aligned} \tag{7}$$

where

$$\begin{aligned} \lambda_1^2 &= k_x^2 + k_y^2 - k_p^2, \quad k_p = \frac{\omega}{c_p} \\ \lambda_2^2 &= k_x^2 + k_y^2 - k_s^2, \quad k_s = \frac{\omega}{c_s} \end{aligned} \quad . \tag{8}$$

The solution of differential equations (7) in the transformed domain should satisfy the Sommerfield's radiation condition, which means that there is no propagation of waves from infinity toward the source. Therefore $A_1 = B_{x1} = B_{y1} = 0$, giving the solutions in the following form

$$\hat{\varphi} = A_2 e^{-\lambda_1 z}, \qquad \hat{\psi}_x = B_{x2} e^{-\lambda_2 z}, \qquad \hat{\psi}_y = B_{y2} e^{-\lambda_2 z} \quad . \tag{9}$$

Substituting Eq. (9) into the Eqs. (5) results in the following relation between the displacement vector $\hat{\mathbf{u}}$ and vector of unknown coefficients $\mathbf{C}$

$$\hat{\mathbf{u}} = \mathbf{A}^u \cdot \mathbf{C} \tag{10}$$

where

$$\hat{\mathbf{u}} = \begin{Bmatrix} \hat{u}_x \\ \hat{u}_y \\ \hat{u}_z \end{Bmatrix}, \quad \mathbf{A}^u = \begin{bmatrix} ik_x & 0 & \lambda_2 \\ ik_y & -\lambda_2 & 0 \\ -\lambda_1 & -ik_y & ik_x \end{bmatrix}, \quad \mathbf{C} = \begin{Bmatrix} A_2 \\ B_{2x} \\ B_{2y} \end{Bmatrix}. \tag{11}$$

The unknown coefficients A_2, B_{x2} and B_{y2} can be obtained from the boundary conditions at the surface of the half space, defined as

$$\begin{Bmatrix} \hat{\sigma}_{zx}(k_x, k_y, z=0, \omega) \\ \hat{\sigma}_{zy}(k_x, k_y, z=0, \omega) \\ \hat{\sigma}_z(k_x, k_y, z=0, \omega) \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ \hat{p}_z(k_x, k_y, \omega) \end{Bmatrix} = \hat{\mathbf{p}}(k_x, k_y, \omega) \tag{12}$$

where $\hat{p}_z(k_x, k_y, \omega)$ is the Fourier's transform of the applied moving load $p_z(x,y,t)$

$$\hat{p}_z(k_x, k_y, \omega) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} p(x,y,t) e^{-i(k_x x + k_y y + \omega t)} dxdydt \tag{13}$$

Using well known relations between stress and displacements [10], the stress in wavenumber domain can be written as.

$$\boldsymbol{\sigma} = \mathbf{A}^{\sigma}\mathbf{C} \tag{14}$$

where

$$\mathbf{A}^{\sigma} = \mu \begin{bmatrix} -2ik_x\lambda_1 & k_xk_y & -\left(\lambda_2^2 + k_x^2\right) \\ -2ik_y\lambda_1 & \left(\lambda_2^2 + k_y^2\right) & -k_xk_y \\ 2k_r^2 - k_s^2 & 2ik_y\lambda_2 & -2ik_x\lambda_2 \end{bmatrix} \tag{15}$$

and $k_r^2 = k_x^2 + k_y^2$.

Substituting solution of the Eq. (14) into the Eq. (10), regarding the Eq. (12), gives the displacement vector in wavenumber domain as

$$\hat{\mathbf{u}}(k_x,k_y,\omega) = \hat{\mathbf{H}}(k_x,k_y,\omega)\hat{\mathbf{p}}(k_x,k_y,\omega)\ , \tag{16}$$

where $\hat{\mathbf{H}} = \left[\mathbf{A}^{u}(\mathbf{A}^{\sigma})^{-1}\right]$ is the transfer function matrix (compliance) of the half-space.

The response in the wavenumber domain demands the transformation in the space-time domain by usage of the inverse Fourier transform

$$f(x,y,t) = \frac{1}{8\pi^3}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\hat{f}(k_x,k_y,\omega)e^{i(k_xx+k_yy+\omega t)}dk_xdk_yd\omega\ . \tag{17}$$

In these evaluations, damping is taken into account by using complex values for the Lamé’s constants according to the principle of correspondence

$$\hat{E} = E\left(1+2i\xi\right), \qquad \hat{G} = G\left(1+2i\xi\right) \tag{18}$$

where ξ is damping coefficient.

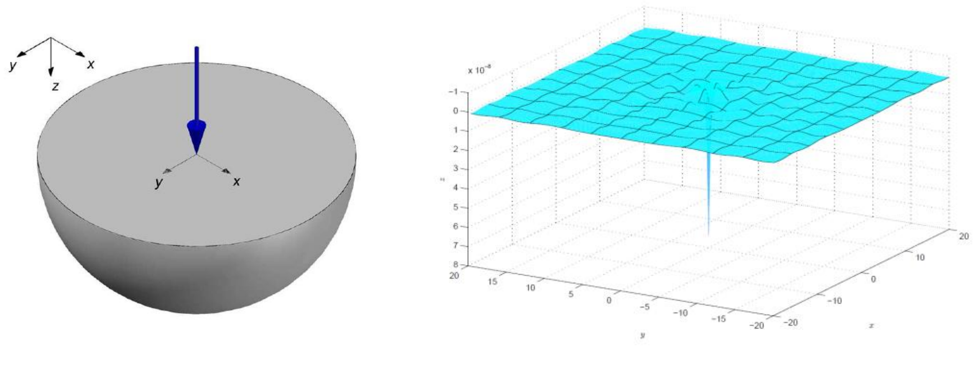
3. Dynamic stiffness of the soil

The foundation is considered rigid, and its dynamic stiffness matrix is obtained from the dynamic stiffness matrix of the flexible foundation using kinematics transformation, [11].

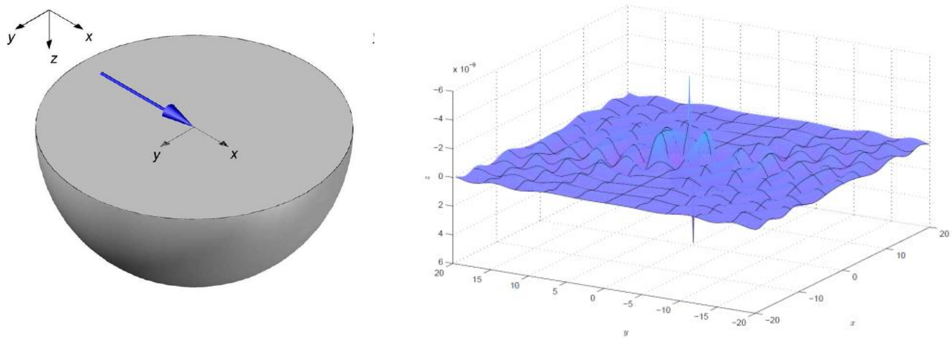
3.1. Dynamic stiffness matrix of flexible rectangular foundation

The dynamic stiffness matrix of the flexible rectangular foundation, $\hat{\mathbf{K}}_{ff}$, at certain frequency ω, is obtained by inverting the dynamic flexibility matrix, $\hat{\mathbf{K}}_{ff}(\omega) = \hat{\mathbf{F}}_{ff}^{-1}(\omega)$. Elements of dynamic flexibility matrix $\hat{\mathbf{F}}_{ff}(\omega)$ represent the nodal displacements at the surface of the half-space due to corresponding harmonic forces of unit amplitude, Fig. 1. They are obtained using ITM. If $n\times n$ is a number of nodes of a rectangular surface on half-space, the dimension of the flexibility matrix, as well as the dynamic stiffness matrix, is $3n\times 3n$. Nodal displacements vector $\hat{\mathbf{u}}_f\,(3n,1)$ and corresponding force vector $\hat{\mathbf{P}}_f\,(3n,1)$ are related by dynamic stiffness matrix of flexible foundation $\hat{\mathbf{K}}_f\,(3n,3n)$

$$\hat{\mathbf{P}}_f = \hat{\mathbf{K}}_{ff}\cdot\hat{\mathbf{u}}_f \tag{19}$$



a)



(*b*)

Figure 1. Vertical displacements u_z (*a*) and horizontal displacement u_x, (*b*) due to unit forces in *z* and *x* direction respectively, ω = 160 *rad*/*s*, ν = 0.4

3.2. Dynamic stiffness matrix of rigid rectangular foundation

Dynamic stiffness matrix of the corresponding rigid, massless, rectangular foundation, Fig. 4, is obtained from the dynamic stiffness matrix of the flexible foundation using kinematics transformation. Rigid foundation has 6 degrees of freedom: three translations in *x*, *y* and *z* directions, and three rotations around *x*, *y* and *z* axes. The displacements vector $\hat{\mathbf{u}}_r$ of the centroid *O* at the interaction surface and corresponding force vector $\hat{\mathbf{P}}_r$ are related by:

$$\hat{\mathbf{P}}_r = \hat{\mathbf{K}}_{fr}\hat{\mathbf{u}}_r, \tag{20}$$

where $\hat{\mathbf{K}}_{fr}$ (6x6) is the dynamic stiffness matrix of rigid foundation, and corresponding vectors are

$$\hat{\mathbf{u}}_r^t = \left\{\hat{u}_x \quad \hat{u}_y \quad \hat{u}_z \quad \hat{\varphi}_x \quad \hat{\varphi}_y \quad \hat{\varphi}_z\right\}, \qquad \hat{\mathbf{P}}_r^t = \left\{\hat{P}_x \quad \hat{P}_y \quad \hat{P}_z \quad \hat{M}_x \quad \hat{M}_y \quad \hat{M}_z\right\}. \tag{21}$$

Vector of nodal displacements $\hat{\mathbf{u}}_f$ and vector $\hat{\mathbf{u}}_r$ relate with kinematics constraint equation

$$\hat{\mathbf{u}}_f = \mathbf{a}\cdot\hat{\mathbf{u}}_r \tag{22}$$

where $\mathbf{a}(3(n\times n),6)$ is kinematics matrix:

$$\mathbf{a}^t = \left\{\mathbf{a}_1 \quad \cdots \quad \mathbf{a}_i \quad \cdots \quad \mathbf{a}_{nxn}\right\}. \tag{23}$$

This matrix consists of $n\times n$ sub-matrices $\mathbf{a}_i$, i=1,2, …, $n\times n$. Each sub-matrix $\mathbf{a}_i$ is obtained by kinematics consideration, regarding node $A = i$ and centroid of foundation *O* (Fig. 2), as

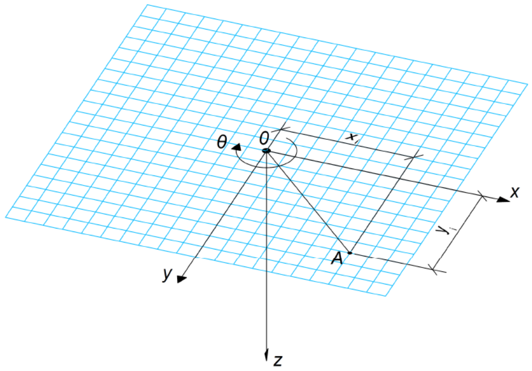


Figure 2. Surface foundation

$$\mathbf{a}_i = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & -y_i \\ 0 & 1 & 0 & 0 & 0 & x_i \\ 0 & 0 & 0 & y_i & -x_i & 0 \end{bmatrix} \tag{24}$$

where x_i and y_i are coordinates of node i.

Equalizing the energy of the deformation for flexible and rigid foundations

$$\hat{\mathbf{P}}_f^T \hat{\mathbf{u}}_f = \hat{\mathbf{P}}_r^T \hat{\mathbf{u}}_r \tag{25}$$

and taking into account Eqs. (19), (20) and (22), obtained is the relation between the dynamic stiffness matrix for rigid and flexible foundation in the form

$$\hat{\mathbf{K}}_{fr} = \mathbf{a}^T \hat{\mathbf{K}}_{ff} \mathbf{a}\,. \tag{26}$$

The dynamic stiffness matrix of rigid rectangular foundation $\hat{\mathbf{K}}_{fr}$ is frequency dependent matrix, which elements are complex. Real part represents the dynamic stiffness, while imaginary part represents damping of the foundation in the certain direction.

3. Displacements of the half-space due to the moving load

The displacement of the half-space due to the moving wheel has been object of investigation of many researchers [12],[13]. The interaction force between the wheels and rail/road can be simulated by a quasi-static term plus dynamic term. In the study of vehicle-induced vibrations the quasi-static term has been frequently modeled by the point load moving with a constant speed along the surface of visco-elastic half-space. The dynamic term could be neglected if the moving force velocity v is less than Rayleigh wave velocity c_R [14]. If $v>c_p$ the influence of dynamic part is significant, especially in the case of soft soil [13].

The displacements in the free-field at the soil-structure interface are obtained due to the vertical point force moving in x-direction along the surface of a half-space with a constant speed v

$$p_z(x,t) = p_o \delta(x - vt) \tag{27}$$

According to the shifting theorem [13] the Fourier transform of the moving force is

$$\hat{p}_z(k_x,\omega) = 2\pi \cdot p_o \cdot \delta(\omega + vk_x)\,. \tag{28}$$

Eq. (28) shows that the results obtained for the case of a stationary loading ($v=0$) can be transferred to the case of a load moving with the velocity v in the x-direction if ω is substituted by $\tilde{\omega} = \omega + k_x v$. The displacement vector at the receiver point S in the space domain is found by substituting Eq. (28) into the Eq. (16) and applying inverse Fourier transform. Taking into account the shifting theorem obtained is final result in the form:

$$\mathbf{u}(x,y,t) = \frac{1}{(2\pi)^3} \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \hat{\mathbf{H}}(k_x,k_y,\tilde{\omega} - k_x v)\hat{\mathbf{p}}(k_x,k_y,\tilde{\omega}) e^{ik_x\bar{x}} e^{ik_y y} e^{i\tilde{\omega}t} dk_x dk_y dt\ , \tag{29}$$

where

$$\bar{x} = x - vt, \qquad \tilde{\omega} = \omega + k_x v\ , \tag{30}$$

represents the moving coordinate system $\bar{x}$ and frequency $\tilde{\omega}$ at the source, respectively.
From the Eq. (28) follows that $\omega = -k_x v$ i.e. $\tilde{\omega} = 0$, which means that integral (29) is constant in time. Therefore, the response of the half-space due to the moving load can be expressed in the moving frame of reference as:

$$\mathbf{u}(\bar{x}, y) = \frac{1}{(2\pi)^2} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \hat{\mathbf{H}}(k_x, k_y, -k_x v) \hat{\mathbf{p}}(k_x, k_y) e^{ik_x \bar{x}} e^{ik_y y} dk_x dk_y \,. \tag{31}$$

The obtained integral is the same as in the case of stationary force, Fig. 1a. The only difference is that in the case of moving load the compliance of the soil has to be calculated with the shifted frequency $\omega = -k_x v$.

5. Spectral element model

In Fig 3. presented is the one storey structure founded on homogeneous elastic half space. The structure is subjected to vertical ground vibrations induced by moving load.

The numerical model consists of two-dimensional plate spectral element, and four one-dimensional spectral elements (columns). The structure is founded on rigid surface footings. The vertical dynamic stiffness of the soil-foundation interface is represented by mutually independent springs, Fig. 4. Since the structure is subjected to vertical ground vibrations, the horizontal vibrations in *x* and *y* direction as well as the rotations about the *z* axis are restrained.

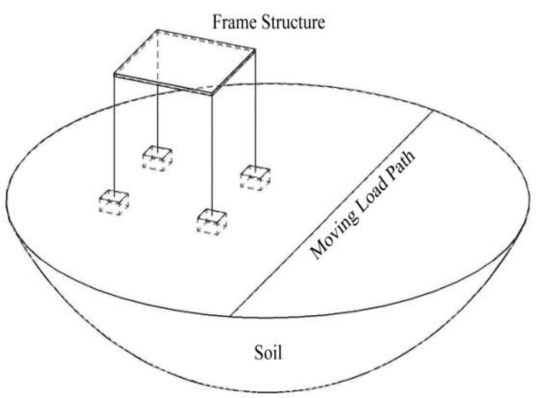


Figure 3. Structure on half-space

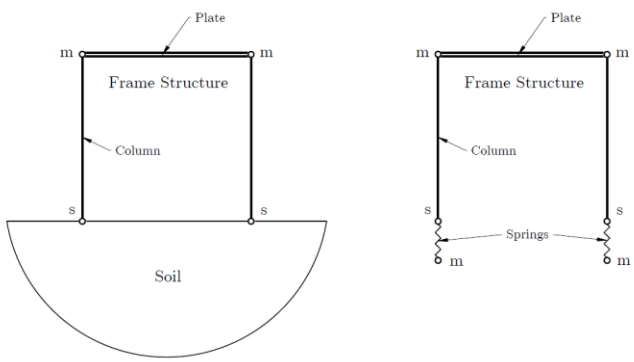


Figure 4. Model with springs

The dynamic response of structure is obtained in frequency domain using the substructure approach and the Spectral Element Method. The columns are modeled with Euler-Bernoulli beam spectral element. The displacement field of beam element is represented by trigonometric functions, obtained as exact solutions of the beam's partial differential equations of motion for axial deformation and bending, respectively, [8]. The plate is modeled with two-dimensional plate element in bending. The plate displacement field is presented as Fourier series, where the number of elements is chosen so that solutions satisfy the governing equation of motion with the prescribed degree of accuracy [15]. The problem of assembling 2D plate spectral element with 1-D column spectral element is solved and presented in Nefovska-Danilovic [15]. The details of spectral element formulation as well as the dynamic stiffness matrices for column and plate elements can be found in literature [8], [9], [15].

The equations of motion of the column – soil springs system are given by:

$$\begin{bmatrix} \hat{\mathbf{K}}_{\mathbf{mm}} & \hat{\mathbf{K}}_{\mathbf{ms}} \\ \hat{\mathbf{K}}_{\mathbf{sm}} & \hat{\mathbf{K}}_{\mathbf{ss}} \end{bmatrix} \begin{Bmatrix} \hat{\mathbf{u}}_{\mathbf{m}} \\ \hat{\mathbf{u}}_{\mathbf{s}} \end{Bmatrix} = \begin{Bmatrix} 0 \\ \hat{\mathbf{P}}_{\mathbf{s}} \end{Bmatrix}, \tag{32}$$

where $\hat{\mathbf{K}}_{\mathbf{ij}}$ i,j=m,s are the stiffness sub-matrices of the column-soil structure, $\hat{\mathbf{u}}_{\mathbf{m}}$ and $\hat{\mathbf{u}}_{\mathbf{s}}$ are the total displacement sub-vectors at the master-**m**, and slave-**s** nodes, respectively, while $\hat{\mathbf{P}}_{\mathbf{s}}$ is the vector of effective force acting on the slave nodes. Master nodes are nodes at the top of the columns, while slave nodes are nodes at interface, Fig. 4.

The effective force vector $\hat{\mathbf{P}}_{\mathbf{s}}$ is obtained by multiplying the dynamic stiffness of rigid foundation with the displacements due to the moving force at interface

$$\hat{\mathbf{P}}_{\mathbf{s}} = \hat{\mathbf{K}}_{fr} \cdot \hat{\mathbf{u}}_{\mathbf{s}} . \tag{33}$$

Eliminating the displacement vector $\hat{\mathbf{u}}_{\mathbf{s}}$ from the equation (32) the relation between the forces and displacements at the master nodes are obtained as:

$$\begin{aligned} &(\hat{\mathbf{K}}_{\mathbf{mm}} - \hat{\mathbf{K}}_{\mathbf{ms}} \hat{\mathbf{K}}_{\mathbf{ss}}^{-1} \hat{\mathbf{K}}_{\mathbf{sm}}) \hat{\mathbf{u}}_{\mathbf{m}} = -\hat{\mathbf{K}}_{\mathbf{ms}} \hat{\mathbf{K}}_{\mathbf{ss}}^{-1} \hat{\mathbf{P}}_{\mathbf{s}}, \\ &\hat{\mathbf{K}}_{\mathbf{m}} \hat{\mathbf{u}}_{\mathbf{m}} = \hat{\mathbf{P}}_{\mathbf{m}} \end{aligned} \tag{34}$$

where $\hat{\mathbf{K}}_{\mathbf{m}} = \hat{\mathbf{K}}_{\mathbf{mm}} - \hat{\mathbf{K}}_{\mathbf{ms}} \hat{\mathbf{K}}_{\mathbf{ss}}^{-1} \hat{\mathbf{K}}_{\mathbf{sm}}$ is the condensed dynamic stiffness matrix of the column – soil springs system and $\hat{\mathbf{P}}_{\mathbf{m}} = -\hat{\mathbf{K}}_{\mathbf{ms}} \hat{\mathbf{K}}_{\mathbf{ss}}^{-1} \hat{\mathbf{P}}_{\mathbf{s}}$ is the condensed force vector, with respect to the master nodes **m**. The matrix $\hat{\mathbf{K}}_{\mathbf{m}}$ is superimposed with the dynamic stiffness matrix of the plate. The result is system of algebraic equations which solution is the vector of unknown displacements $\mathbf{u}_{\mathbf{m}}$ in the master nodes, [15]. The displacement vector is obtained in the frequency domain, so all displacements should be transferred in the time domain using the IFFT. Application of the procedure is presented in the following section.

6. Numerical Example

The top view of the numerical model is shown in Fig. 5, where the foundations F1-F4 and plate contours 1-4 are denoted. The dimensions of plate are *5.0x5.0x0.15 m*. The columns are *3.0 m* high, with rectangular cross section *0.4x0.4 m*. The dimensions of rectangular footings are *1.0x1.0 m*. Material properties of the structure and the soil are given in Table 1.

Table 1. Material properties

	Structure	Soil
Modulus of elasticity, E [kNm^2]	3x10^7	2.08x10^5
Poisson's ratio, υ	0.15	0.30
Density, ρ [kg/m^3]	2500	2000
Damping ratio, ξ	0.05	0.02

The moving load path is parallel to the *x* axis, 5.0 m away from the line defined by the centroids of foundations *F3* and *F4*. The amplitude of the moving load is 1.0 MN. The load is moving with constant speed of 50 km/h. The assumed velocity of the moving force is equal to the speed limit at the urbane zones.

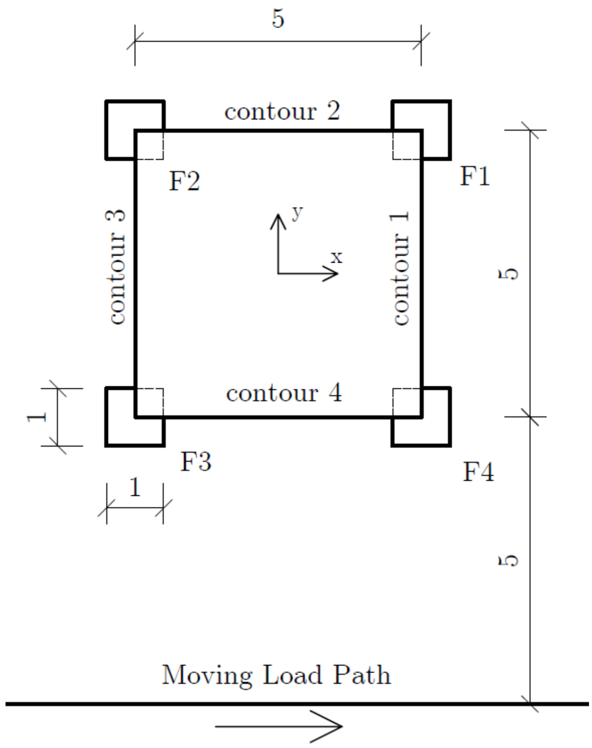


Figure 5. Top view of the numerical model

The dynamic response of the structure was calculated using a computer code developed in Matlab [16], for the range of frequencies from 0.1 to 50 Hz with the frequency step of 0.1 Hz. The impedance functions of the square, massless foundation, as well as, the displacements in the free field at the centroid of each foundation were calculated for the range of frequencies: 0.1, 1.0, 5.0, 10.0, 20.0, 30.0, 40.0 and 50.0 Hz. The values of impedance functions between chosen frequencies were obtained by interpolation. The obtained results are presented in Figs 6, 7 and 8.

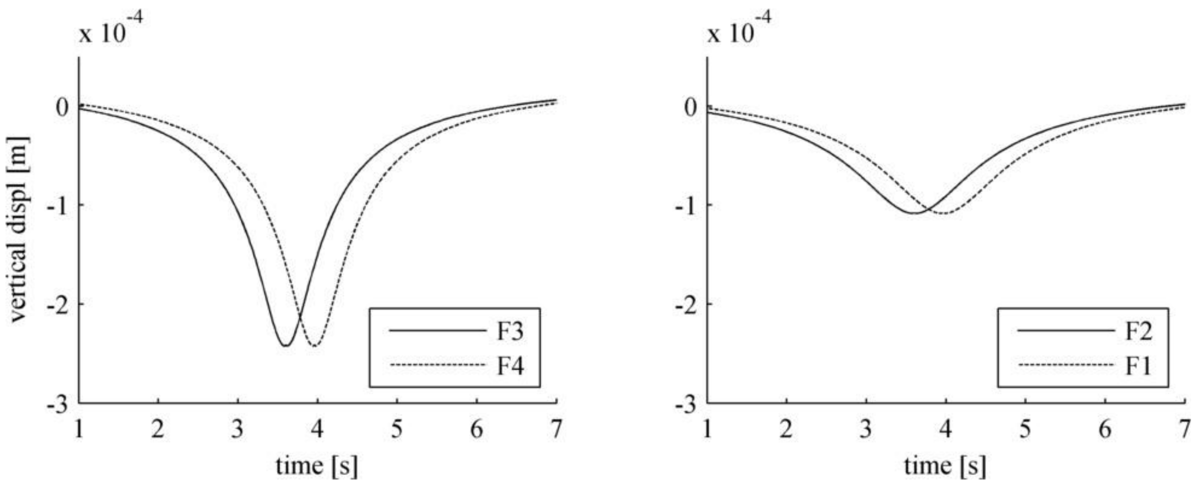


Figure 6. Vertical displacements at the interaction nodes

Fig. 6 shows vertical displacements in the free field at the points of interaction due to the moving load. The displacements demonstrate quasi-static character with the time. One can notice that displacements in the point F3 are equal to displacements in F4, shifted by the *time delay,* i.e. time that force needs to cross the path from the point F3 to the point F4. The displacements in F1 and F2 are less than displacements in F3 and F4.

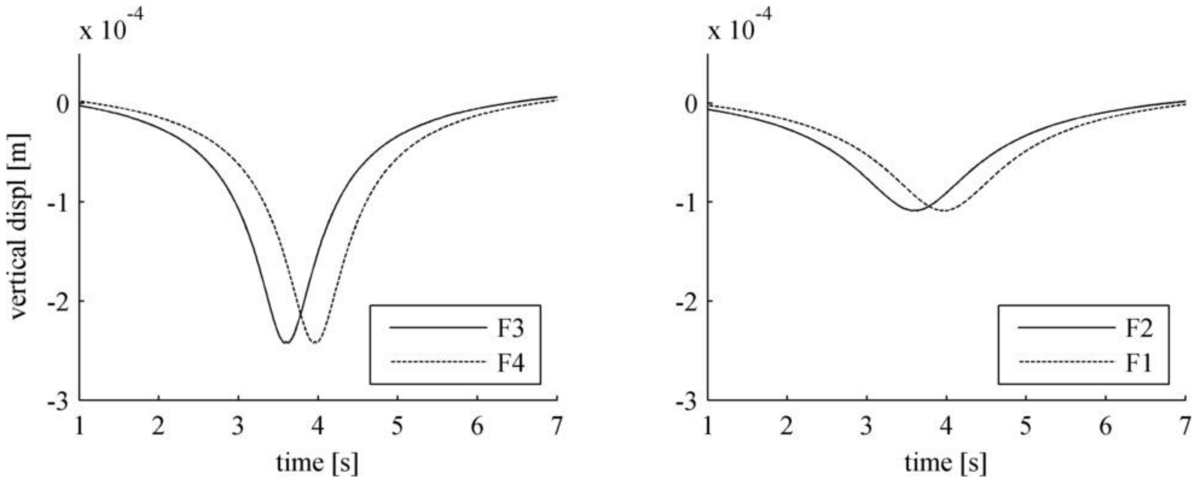


Figure 7. Vertical displacement at the top of the columns

Figs. 7 and 8. show vertical displacements time history at the corners (top of the columns) and at the midpoint of the plate, respectively. The displacements at the top of the columns are almost the same as the displacements at the soil-structure interface, due to the high stiffness of the columns.

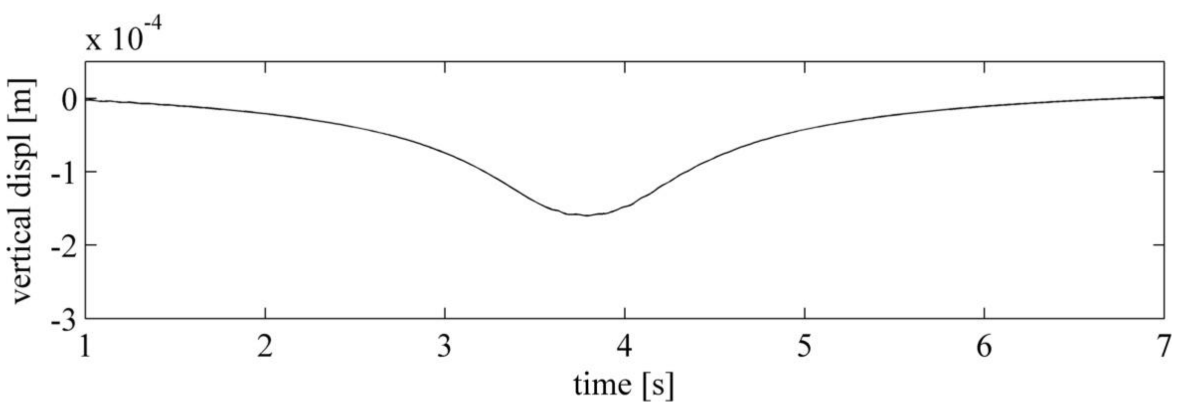


Figure 8. Vertical displacement at the mid-point of the plate

Maximum displacements at the characteristic nodes are presented in Table 2. The maximum displacement at the midpoint of the plate is less than maximum displacements at the plate corners which correspond to the nodes F3 and F4, but higher than displacements at the corners which correspond to the nodes F1 and F2.

Table 2. Maximum displacements at the characteristic points

Position	Max displacement	Time	Position	Max displacement	Time
interface	[mm]	[s]	plate	[mm]	[s]
F1	1.083E-04	3.924	F1	1.086E-04	3.924
F2	1.083E-04	3.564	F2	1.086E-04	3.564
F3	2.424E-04	3.582	F3	2.423E-04	3.582
F4	2.424E-04	3.942	F4	2.423E-04	3.942
			midpoint	1.597E-04	3.780

7. Conclusions

The dynamic response of structure to ground vibrations caused by moving force is calculated in the frequency domain using the substructure approach. The dynamic interaction between the structure and sub-grade is solved using total displacements formulation. The structure is modeled by spectral beam and plate elements. The impedance functions of the half-space, as well as, the displacement of foundations due to moving force are obtained using ITM. A computer code has been developed for the analysis of soil dynamics and the vibration analysis of plate-column-soil assemblies using Matlab.

Since the vertical vibrations are more perceptible by building occupants, in the given example the effect of moving force on dynamic response of structure is shown for vertical excitation only. The foundation displacements as well as the displacements of the structural nodes induced by the moving force, demonstrate quasi-static character. The results show that proposed methodology can be used to solve such problem.

This formulation can be extended and applied to multi-storey building resting on layered sub-grade. Also, the horizontal components of the displacement vectors at the interaction nodes can be taken into account. It means that in-plane vibrations of plate have to be included in the analysis.
In order to represent the moving vehicle, the system of moving forces should be considered. The dynamic component can be taken into account by multiplying the moving force with exponential function that represents a moving wheel and road/truck imperfection.

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