

# Comparison Between the Linear and Nonlinear Responses of Cable Structures I – Static Loading

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## Abstract

*Cable suspended structures undergo large displacements under loading. This paper analyzes the differences between the linear and nonlinear responses of cable suspended structures under static loading. Several types of structures (cable truss, hyperbolic paraboloid) are being studied in order to make an explicit comparison between the linear and nonlinear responses. Multiple levels of pretension are considered varying from 5% to 50%.*

## Rezumat

*Structurile suspendate pe cabluri sunt structuri pretensionate care prezintă deplasări mari sub încărcări. Lucrarea analizează diferențele dintre răspunsul liniar și cel neliniar al structurilor suspendate pe cabluri supuse la încărcări statice. Sunt studiate mai multe tipuri de astfel de structuri (fermă cablu, structură sub formă de paraboloid hiperbolic) pentru a realiza o comparație explicită între cele două răspunsuri (liniar și neliniar). Se consideră mai multe nivele de pretensionare între 5% și 50%.*

**Keywords:** cable structures, linear response, nonlinear response

## 1. Introduction

The nonlinear response of cable structures is widely discussed in literature stressing that a nonlinear analysis is necessary in order to determine their accurate behaviour. But no explicit reference is being made to the actual differences between the linear and nonlinear responses. In this paper an explicit comparison of the two responses is being made. Two structures were subjected to linear and nonlinear analyses using NELSAS [1] and SAP2000 [2] softwares. The variable parameters in the study are considered the pretensioning force and the loading. The loads are equal concentrated loads and they increase in value until they reach the proximity of the slackening load.

In practice, cable structures are designed to withstand loads without slackening. Thus, under the action of the maximum probable load, at least some pretension will remain in all of the structural members.

The results are given in the form of graphs and tables which contain the values (in percents) representing the differences between the linear and nonlinear responses. Differences in displacements and axial forces are being considered.

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## 2. Nonlinearity

Most engineering structures are considered to behave linearly (their analysis is based upon a linear relationship between forces and displacements). However there are exceptions where the linear relationship between forces and displacements cannot describe correctly the behaviour of the structure, which is nonlinear. The sources of nonlinearity are primarily due to:

- nonlinear behaviour of the material
- nonlinear geometric behaviour
- or both effects combined.

Cable structures exhibit ‘flexibility’ thus determining the nonlinear geometric behaviour. “We should (...) attribute the ‘flexibility’ of cable networks not to the low axial stiffness of the constituent cables (which is often not true), but to the geometry of the structure” [6]. In this paper the material is considered to behave linearly and only the geometric nonlinearity is being accounted for (it being dominant for cable suspended structures).

## 3. Description of analysis used

The example structures were analyzed using NELSAS[1] (Non-linear static analysis, in finite deformation, of cable and pin-jointed bar structures) and SAP2000[2] softwares. In NELSAS software the cables were modeled using straight elements connected in nodes. The catenary cable element was used in SAP2000 software. Linear and nonlinear static analyses were run for both cable structures considered. For the nonlinear analyses an iteration convergence tolerance of  $1E-6$  was used. The structures were modeled and introduced in the analysis softwares in their equilibrium configurations under prestress. The self weight of the cables was neglected.

The loads were defined as concentrated loads applied in the nodes of the structures.

## 4. Example structures

The biconcave cable truss (Fig. 1) is a plane truss with vertical hangers and a span of 70,00 m. It is made up of ten 7,00 m panels. The total height of the truss is 11,35 m with a height of 2,00 m in the middle of the span. The cables are considered to have a circular cross section with sectional areas of  $27,8256 \text{ cm}^2$ ,  $20,0862 \text{ cm}^2$  and  $0,5969 \text{ cm}^2$  for the sagging, hogging and hanger cables. The modulus of elasticity of the cables is  $16500 \text{ kN/cm}^2$ . The truss is supported by 4 pin-joints and has a total of 36 degrees of freedom. The geometrical configuration was established according to [3] and cable characteristics according to [4].

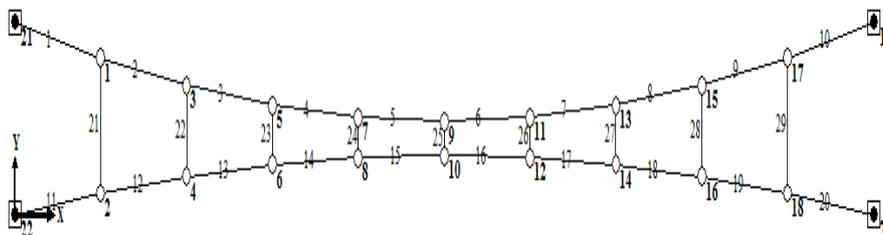


Figure 1. Biconcave cable truss [NELSAS].

The hyperbolic paraboloid [5] (Fig. 2) is a 35,052 x 35,053 m structure, known in literature as the Aden Airways building. It has the shape function:

$$y = \frac{5.334}{17.526^2} z^2 - \frac{3.81}{17.526^2} x^2$$

and has seven cables each way in equal spacing. The cables have  $EA=210,312$  MN and the horizontal component of cable prestress tension is 200,17 kN for cables in the  $x$ -direction and 142,97 kN for cables in the  $y$ -direction. The structure has 75 degrees of freedom.

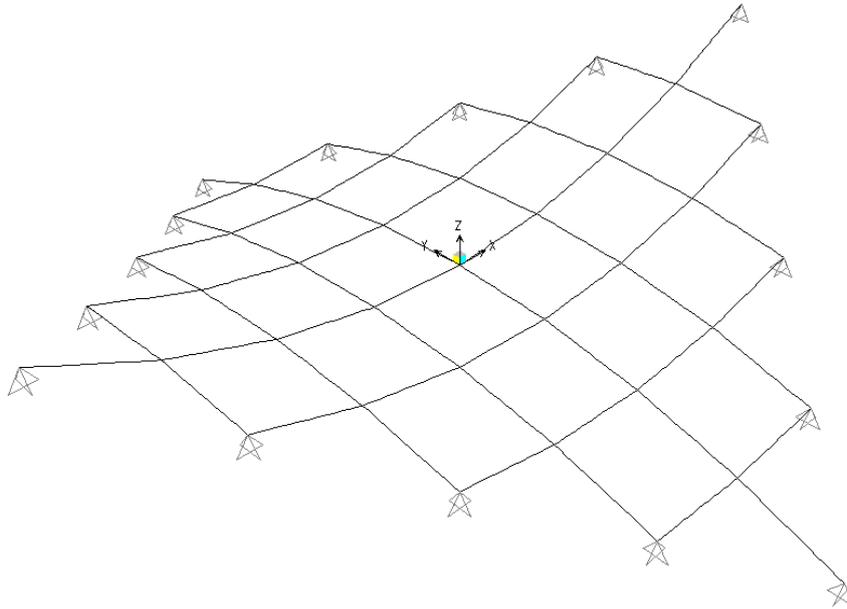


Figure 2. Hyperbolic paraboloid [SAP2000].

The cables are considered to have circular cross sections with sectional areas of  $12,71 \text{ cm}^2$ .

## 5. Results

### 5.1 The cable truss

Two loading patterns were considered for the cable truss. Firstly, the truss was loaded with the same loads at different levels of pretension (The maximum value of the loading interval corresponds to the maximum load that doesn't slacken the structure at the lowest pretension force). Secondly, for each case of pretensioning the loads increased up to the maximum value that doesn't slacken the structure. The pretensioning of the cable truss was varied from 20% to 50%. 15 load cases (steps) were considered. The allure of the load-displacement graphs is presented in figures 3 and 4. Figure 3 contains the displacements measured from the initial equilibrium configuration of the structure whereas figure 4 shows the variation of displacements from one loading step to the next. The values on the ordinates are the displacements in meters.

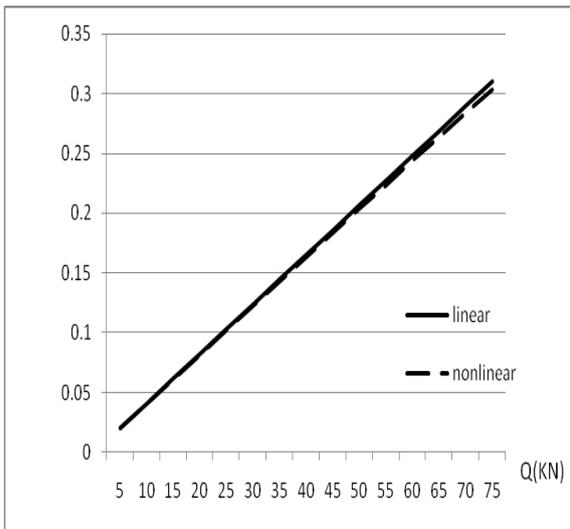


Figure 3. Load-displacement curves.

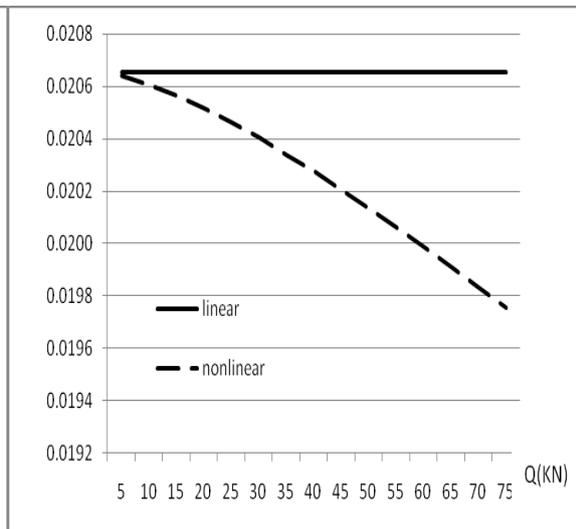


Figure 4. Load-displacement curves.

In table 1 the differences between displacements are given for the first loading pattern:

Table 1.

Pretension	Loading case	%*
20%	5	0.463
	10	1.157
	15	1.973
30%	5	0.606
	10	1.338
	15	2.146
40%	5	0.674
	10	1.426
	15	2.229
50%	5	0.708
	10	1.469
	15	2.264

$$* \% = \frac{|L-N|}{L} \times 100$$

L- displacement after linear analysis

N- displacement after nonlinear analysis

For loads that precede the slackening of the structure considering all levels of pretension separately the results are given in table 2:

Table 2.

Pretension	%
20%	1.99
30%	3.36
40%	4.06
50%	5.01

The results given in tables 1 and 2 are those for a central node (no. 10). In table 2 only the maximum differences are given (those corresponding to loading case 15).

The allure of the load-axial force curves is presented in figures 5, 6 and 7 for three kinds of structural members (elements from the sagging cable, hogging cable and hangers). The values on the ordinates are the axial forces in kN.

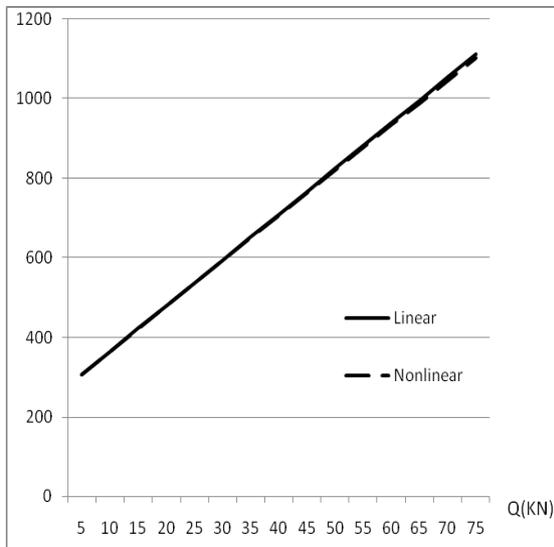


Figure 5. Load-axial force curves (sagging cable element).

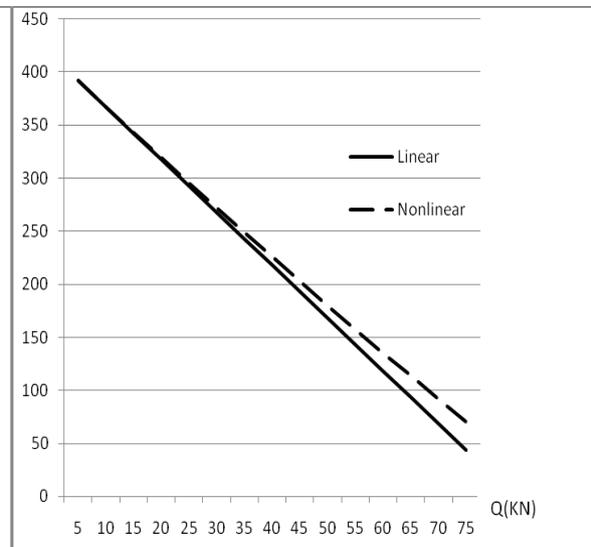


Figure 6. Load-axial force curves (hogging cable element).

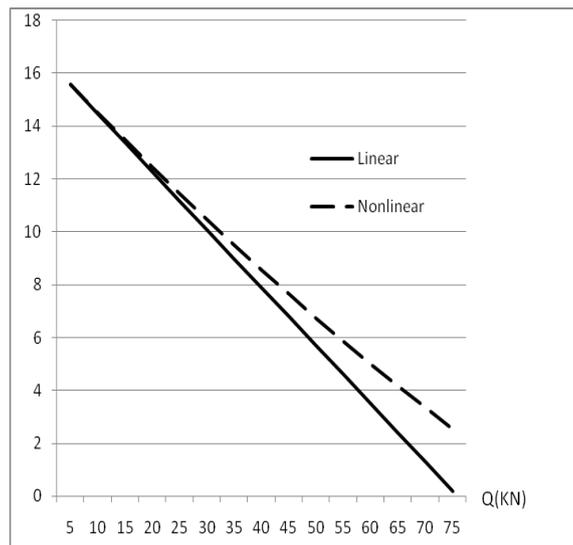


Figure 7. Load-axial force curves (hanger).

### 5.2 The hyperbolic paraboloid

For the hyperbolic paraboloid the same loading patterns were considered for two pretension cases. The first case is the one found in literature which corresponds to a pretension of 10% and the second is a pretension of 5%. If the displacements measured from the initial equilibrium configuration of the structure are considered, two patterns of linear/nonlinear load-displacement

curves can be identified (Figures 8 and 9). The values on the ordinates are the displacements in meters.

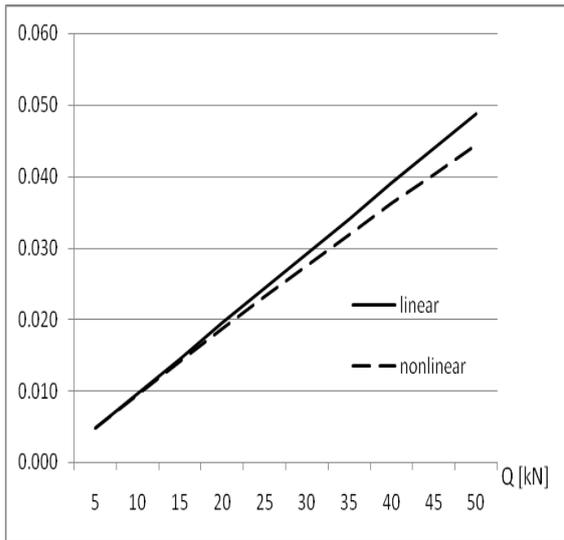


Figure 8. Load-displacement curves (1) (hyperbolic paraboloid).

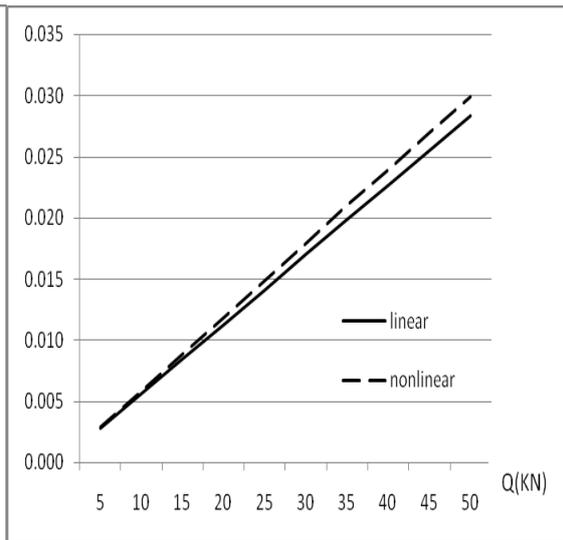


Figure 9. Load-displacement curves (2) (hyperbolic paraboloid).

The differences in percents between displacements in the linear/nonlinear analyses are given in table 3:

Table 3. Results for the hyperbolic paraboloid (displacements).

Pretension	Loading pattern	
	1	2
5%	0-6.04	0-15.7
10%	0-6.04	0.29-11.37

The differences in percents between axial forces in the linear/nonlinear analyses are given in table 4:

Table 4. Results for the hyperbolic paraboloid (axial forces).

Pretension	Loading pattern	
	2	
	sagging c.	Hogging c.
5%	0-3.72	0-20.51
10%	0-5.27	0-28.67

## 6. Conclusions

Two cable suspended structures, cable truss and hyperbolic paraboloid, were analyzed using linear and nonlinear analyses. The differences between displacements go up to 5% for the cable truss and 15,7% for the hyperbolic paraboloid. For axial forces, significant differences occur in the last quarter of the loading interval but only in the structural members where the pretension decreases

under loading (hogging cables, hangers). The differences between axial forces for sagging cables are similar to the differences between displacements.

It was observed that in the case of the hyperbolic paraboloid the displacements can be greater or smaller in the linear or nonlinear analyses depending on the node and loading case (the linear response curve can be above or under the nonlinear response curve, see Figures 8 and 9). The results of the comparison show *small* differences between the linear and nonlinear response of cable structures. Taken into consideration the safety factors provided by codes it can be concluded that a linear analysis is sufficient for the design of prestressed cable structures subjected to static loads.

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## 7. References

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