

## STUDYING THE VARIABLES REQUESTS FOR THE RAILS

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### Abstract

*This paper presents an analytical method for calculating the change in normal uniform effort at a point on the railway track section. For the representative type of convoy considered, it is presented the bending moment variation in section 0 until the train passes through the interval  $[-6L, 6L]$  distance on which is considered that the function  $\mu(x)$  has significant ordered (large enough to be taken into account). In order to obtain high accuracy results we have worked with the program MATHCAD, in double precision, considering a displacement step of 10 cm between two successive positions of the convoy.*

### Rezumat

*În această lucrare este prezentată o metodă analitică de calcul a variației eforturilor unitare normale într-un punct pe secțiunea șinei de cale ferată. Pentru tipul de convoi reprezentativ considerat, este prezentat variația momentului încovoietor în secțiunea 0 până când trenul parcurge intervalul  $[-6L, 6L]$ , distanță pe care se consideră că funcția  $\mu(x)$  are ordonate semnificative (destul de mari pentru a fi luate în considerare). În scopul obținerii unui grad ridicat de precizie a rezultatelor s-a lucrat cu programul MATHCAD în dublă precizie, considerându-se un pas de deplasare de 10 cm între două poziții succesive ale convoiului.*

**Keywords:** effort, stresses variables, intensity, rail, sleeper

### 1. Introduction

For the considered convoy it was developed a calculation programme that contains the input data required for the determination of  $\lambda$  and  $L$  (characteristics of the superstructure) axle weights in (kN) and all axle positions relative to the first axle train. The calculating formula of the bending moment is a function of "i" and "j", where "i" is the number of train axle and "j" is the train position index for which the moment is calculated. Were determined maximum end minimum bending moment, amplitude, area chart Maximum positive and negative efforts were also calculated, efforts which were reached in the rail in the upper and lower fiber in the convoy. For highlighting of variation effort to rail were made suggestive graphics.

## 2. Calculation method

This calculation uses the fictive beam method for determining the maximum bending moment. Calculation scheme is presented in figure 1:

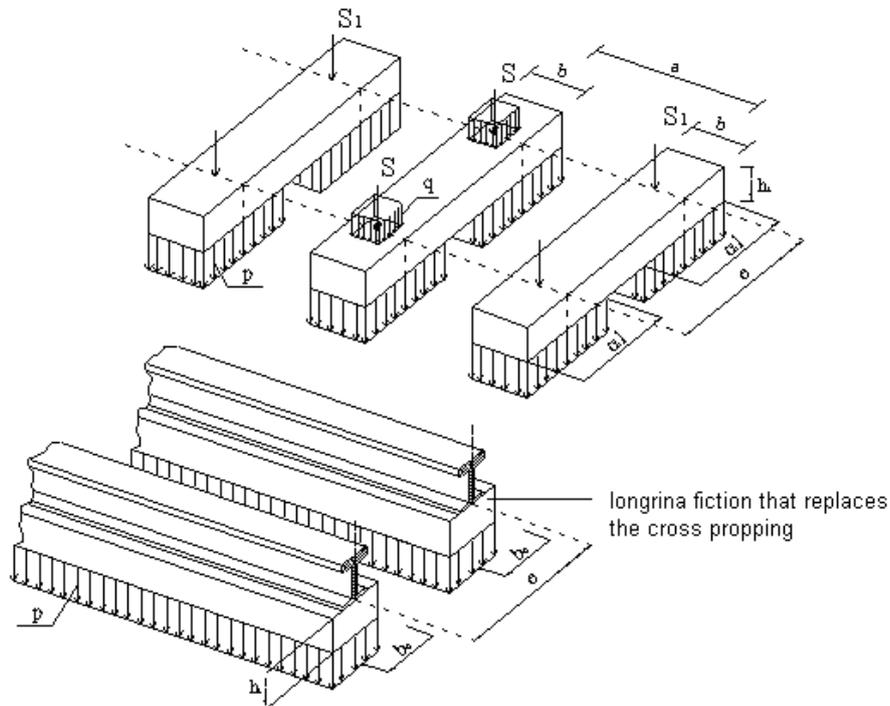


Figure1. Calculation scheme rail

Looking at the request of the railway track it can be seen that during the passage of a convoy (train), the section requires positive bending moments (they stretch lower fiber) and negative bending moments (which extend the upper fiber) considering on the convoy position to the section (figure 2).



Figure2. Schematic representation of positive and negative bending moments occurred in a section of rail from a passing convoy

So for a period of time, bending moment variation in a section is presented in figure 3:

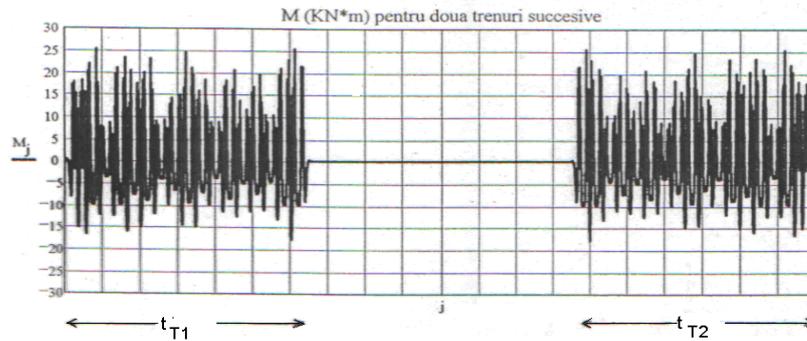


Figure 3. Bending moments versus time in a certain section during the passage of two successive trains [2]

$t_{T1}, t_{T2}, \dots, t_{Tn}$  are the time intervals for a certain section affected by the passage of trains T1, T2, ... Tn.

Functions  $f_1(t), f_2(t), \dots, f_n(t)$  are called specific achievements and the achievements characterized the rail request in section studied.

The life of the rail depends on these functions (specific developments), their frequency for a longer period of time, the shape function, their variations, the maximum expansion of effort and frequency, etc.

To study these functions need of measurements in the rail that requires very exacting and expensive equipment for a long time.

We are considering that between the unitary effort  $\sigma$  and bending moment  $M$  Navier's relationship exists (if the calculation is made in the elastic domain).

$$\sigma_s = \frac{M \cdot z_s}{I_y} = \alpha_s \cdot M \quad \sigma_i = \frac{M \cdot z_i}{I_y} = \alpha_i \cdot M \rightarrow \sigma_{s,i} = \alpha_{s,i} \cdot M \quad (1)$$

Random requests produced in a section of rail during the day, for several days (k days) are shown in Figure 4

Intensity time variation  $\sigma_i(t)$  while recording a sequence is called realization. For assemblies of "k" achievements illustrated in figure 4 which are considered representative for the random request  $\{\sigma(t)\}$  it can be defined at a given time an instant  $t_1$  average of the intensity of the order request first time called.

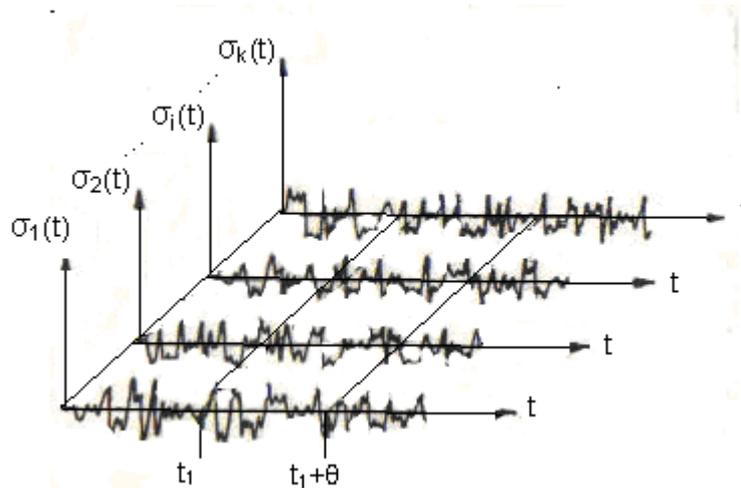


Figure 4. Registration scheme achievements a random requests [1]

$$M_{\sigma} \cong \frac{1}{k} \cdot \sum_{i=1}^k \sigma_i(t_1) \quad [1] \tag{2}$$

Similarly one can highlight the link between the intensity of the request at different times, assessing the autocorrelation function.

$$\mathfrak{R}_{\sigma} \cong \frac{1}{k} \cdot \sum_{i=1}^k \sigma_i(t_1) \sigma_i(t_1 + \theta) \quad [1] \tag{3}$$

This interpretation is important in defining the characteristics and functions of rigorous mathematics associated with random requests. In practice evaluating the characteristics of the random requests through direct application of all outputs, implies the knowledge of a sufficient number of records to substantiate the required statistic consistence which is necessary in the mediation between relations 2 and 3. There is another way to describe the characteristics of a stationary random request, performing the mediation operation in relation to time.

In describing the time, it is considered a particular achievement in the interval [0, T] which defines the average time.

$$m_{\sigma(i)} = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T \sigma_i(t) dt \quad [1] \tag{4}$$

and temporal autocorrelation function:

$$\mathfrak{R}_{\sigma}(\theta, i) = \lim_{T \rightarrow \infty} \frac{1}{T} \cdot \int_0^T \sigma_i(t) \sigma_i(t + \theta) dt \quad [1] \tag{5}$$

If the random request (t) is stationary and the values of "m" and "R" are invariant when mediation in relations (4) and (5) it performs on different realizations (t) is defined as ergodic random request. In this hypothesis (ergodic hypothesis) temporal characteristics are identical with the corresponding features defined on all outputs (2) and (3).

Thus, for a random ergodic request (t) with accomplishments (t) are fulfill the following relations for each of the "i" achievements:

$$m_{\sigma(i)} = m_{\sigma} \quad [1] \tag{6}$$

$$\mathfrak{R}_{\sigma(\theta, i)} = \mathfrak{R}_{\sigma} \quad [1] \tag{7}$$

In this case any particular realization  $\sigma_i(t)$  considered on a sufficiently long period is representative random request  $\{\sigma(t)\}$  as a whole. For the studied section of a railway tracks, a train passes can lead to a representative achievements if trains (convoys) are chosen properly. To establish the function by analytical way the calculation scheme of the rail is adopted, scheme which is represented in figure 1. Bending moment in the rail is calculated by the relationship:

$$M_s = \frac{2 \cdot a \cdot E_s \cdot I_s}{4 \cdot (2 \cdot a \cdot E_s \cdot I_s + \alpha \cdot l_t \cdot E_t \cdot I_t)} \cdot \sqrt[4]{\frac{4 \cdot (2 \cdot a \cdot E_s \cdot I_s + \alpha \cdot l_t \cdot E_t \cdot I_t)}{\alpha \cdot l_t \cdot b \cdot C}} \cdot G \cdot \mu(x) \quad [2] \quad (8)$$

note:

$$m = 2 \cdot a \cdot E_s \cdot I_s \quad (9)$$

$$n = \alpha \cdot l_t \cdot E_t \cdot I_t \quad (10)$$

$$r = \alpha \cdot l_t \cdot b \cdot C \quad (11)$$

Bending moment is:

$$M_s(x) = \frac{m}{4 \cdot (m+n)} \cdot \sqrt[4]{\frac{4 \cdot (m+n)}{r}} \cdot G \cdot \mu(x) \quad [2] \quad (12)$$

note:

$$\lambda = \frac{m}{4 \cdot (m+n)} \cdot \sqrt[4]{\frac{4 \cdot (m+n)}{r}} \quad (\text{characteristic superstructure}) [2] \quad (13)$$

$$M_s(x) = \lambda \cdot G \cdot \mu(x) \quad [2] \quad (14)$$

In the case of several concentrated forces  $G_i$ :  $G_i = \{G_1, G_2, \dots, G_n\}$ , the point O is the bending moment  $M_s(0)$  (figure 5):

$$M_s(0) = \lambda \cdot \sum_{i=1}^n G_i \cdot \mu(x_i) = \lambda \cdot \sum_{i=1}^n G_i \cdot e^{-\frac{x}{L}} \cdot (\cos \frac{x_i}{L} - \sin \frac{x_i}{L}) \quad [2] \quad (15)$$

$\mu(x)$ - is the function representing the bending moment influence line;

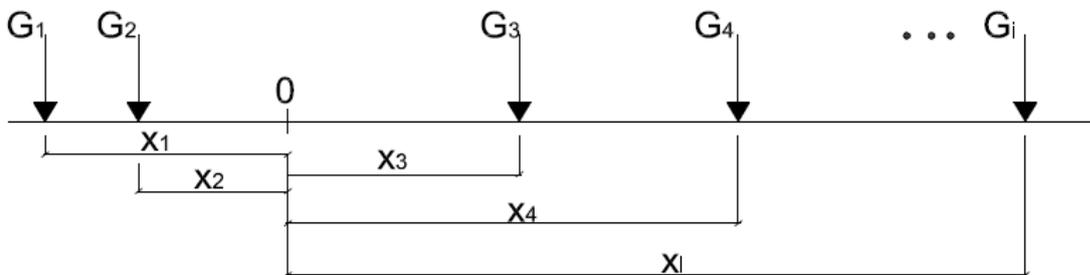


Figure 5. Calculation of bending moment diagram for multiple concentrated forces [2]

The force system moves on the load line with speed  $v$  (m/s) and in different positions produces in section O positive and negative bending moments (figure 6).

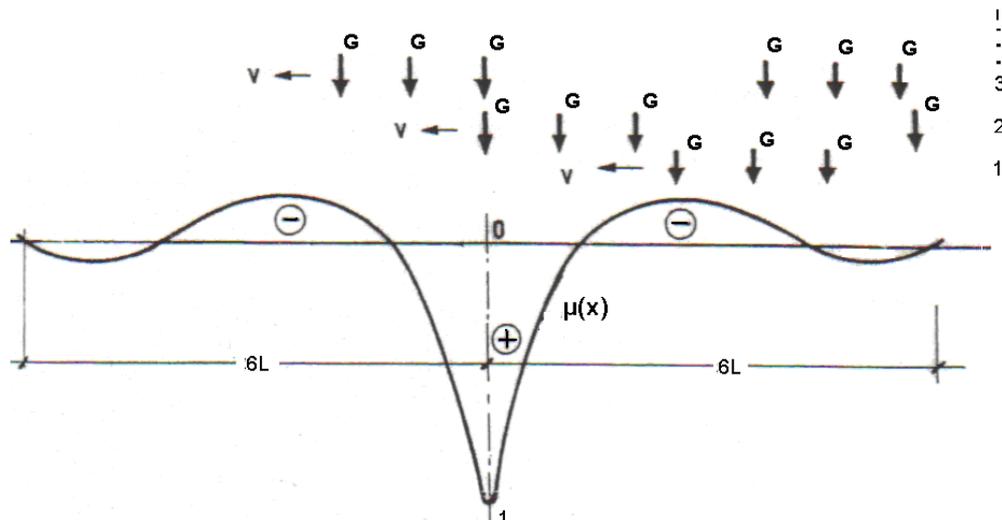


Figure 6. Bending moment influence line [2]

If you represent in a system of axes  $[M0t]$  the bending moments  $M$ , at different time intervals  $\Delta t$ , corresponding to different positions of power system  $G_i$ , we obtain a system with positive and negative points (figure7).

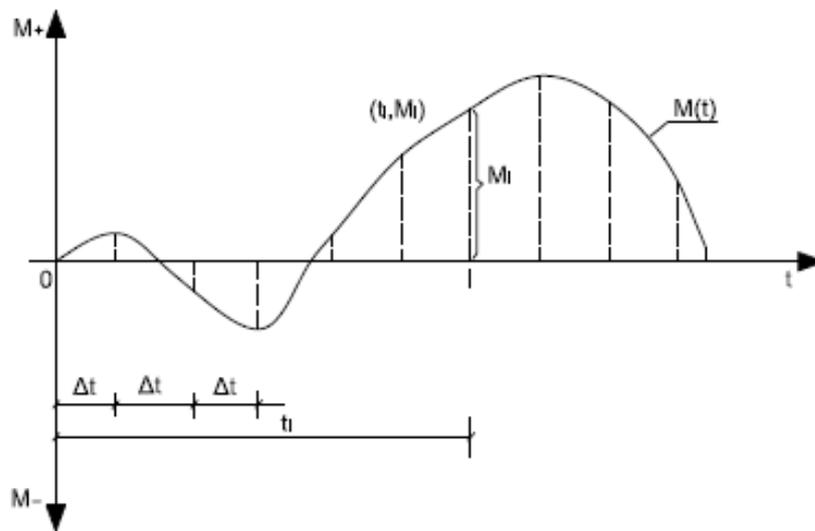


Figure 7. Bending moment variations at different time intervals [2]

Calculation functions  $M(t)$  (the variation of the bending moment in the rail in a section) browsing corresponding interval  $[-6L, 6L]$  by the representative convoy is given below.

For calculation we consider a freight train with forces arrangement as shown in figure 8.

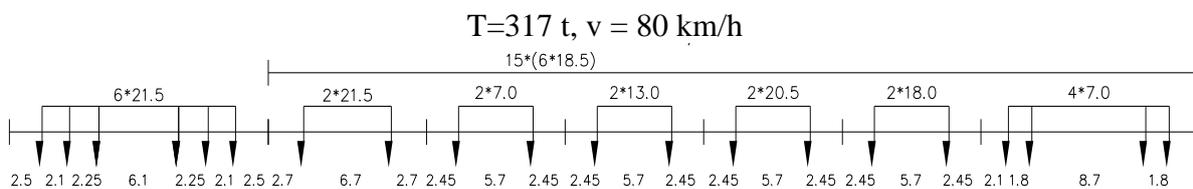


Figure 8. General freight train.

### 3. Characteristics of rail UIC 60E, T17 monoblock sleeper, convoy and superstructure

It features rail UIC 60:

$$E_s=210000 \text{ N/mm}^2; I_s= 3055 \text{ cm}^4; W_s=335 \text{ cm}^3; W_i=377 \text{ cm}^3;$$

$E_s$ -longitudinal modulus of the rail;

$I_s$  - moment of inertia;

$W_s, W_i$  - modulus of the fiber top rail to bottom respectively;

Features monobloc sleeper:

$$a = 65 \text{ cm}; b= 27.8 \text{ cm}; l_t= 260 \text{ cm}; E_t= 37000 \text{ N/mm}^2; I_t=15035 \text{ cm}^4; \alpha= 0.86;$$

$a$  - distance between the rails;

$b$  - width of beam;

$l_t$  - length of beam;

$E_t$  - modulus of elasticity of the beam;

$I_t$  - the moment of inertia of the beam (longrina fictitious);

$\alpha$  - coefficient taking into account material sleepers;

Features convoy:

$$v= 80 \text{ km/h}; \phi = 1.143; L_{\text{train}}=86.2\text{m};$$

$G$  - axle load;

$v$  - speed train;

$\phi$  - coefficient that depends on train speed;

$L_{\text{train}}$ -length train;

Superstructure features:

$$C = 0.03 \text{ N/mm}^3; \delta = 0.3; t= 3; \xi = 1 + t \cdot \delta \cdot \phi; L= 1.453 \text{ m};$$

$C$ -coefficient of bed;

$\delta$  - coefficient of condition of the superstructure;

$t$  - coefficient of probability which depends on the probability of overcoming the efforts in the way;

$L$  – length equivalent.

The values of bending moment, normal unit effort, amplitude, area diagram and general freight train media snapshots are presented in table1. Characteristics of random request from a general freight train.

Table 1

Characteristics of random request	general freight train ( $v=80 \text{ km/h}, G_{\text{axle}}=70..215\text{kN}$ )	
$M_{\text{max}} (+)$ (kNm)	31.92	
$M_{\text{max}} (-)$ (kNm)	13.75	
$\sigma_{s,\text{max}} (+)$ ( $\text{N/mm}^2$ )	95.11	
$\sigma_{i,\text{max}} (+)$ ( $\text{N/mm}^2$ )	84.57	
$\sigma_{s,\text{min}} (-)$ ( $\text{N/mm}^2$ )	40.97	
$\sigma_{i,\text{min}} (-)$ ( $\text{N/mm}^2$ )	36.43	
	upper fiber.	lower fiber
Ampl. ( $\text{N/mm}^2$ )	136.08	121.00
$A$ ( $\text{m}^2$ )	0.089	0.076
$m_\sigma$ ( $\text{N/mm}^2$ )	0.019	0.016

Defining the calculation elements:

$Ampl = |\sigma_{max} - \sigma_{min}|$  - maximum amplitude ( $N/mm^2$ );

A - area diagram between  $\sigma(t)$  (OY axis) and abscissa (Ox axis) ( $m^2$ );

$m_{\sigma}$  - average time ( $N/mm^2$ );

The above values were obtained for random load wagons (General freight train:  $G_{axle} = 70 \dots 215$  kN).

#### 4. Convoy load graphics: general freight train

In the graphic below (figure 9) is represented the bending moment variation in the rail in (kNm) for 1000 successive positions of the train, each position "j" representing a movement of 0.1m train. Train travels  $j = 1000, 0.1m$  (100m) with 80km/h, which is 4.5 seconds. Each horizontal scale represents 100 successive positions of the train (10m) and it is covered in 0.45 seconds.

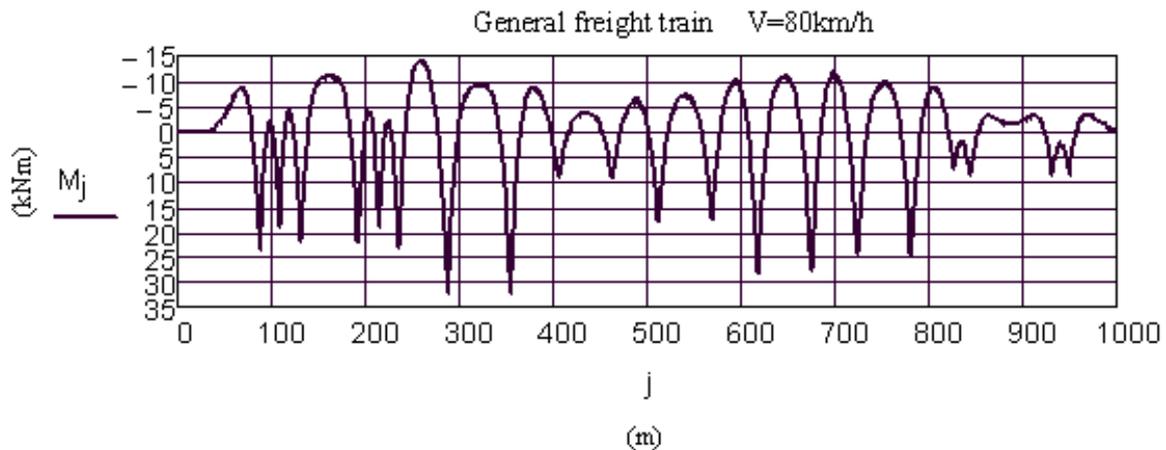


Figure 9. Bending moment variation in the rail according to the position of the train

If the 4.5 seconds the train makes through the 100m to 80km/h are divided into 45 intervals we obtain the graphic below (figure10), where each note on the x-axis represents 0.1 seconds and corresponds to a 2.222m movement of the train.

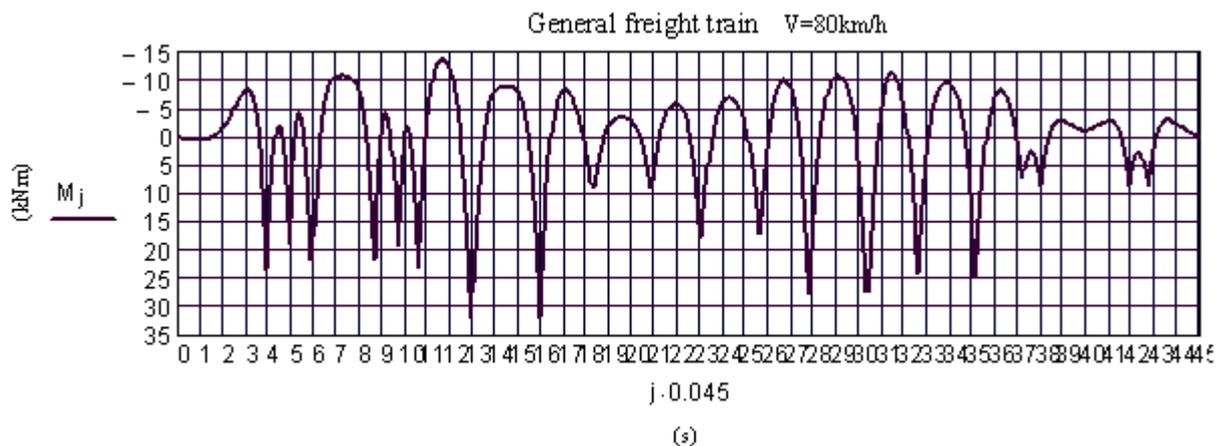


Figure 10. Variation of the bending moment in the rail versus time

Considering  $M(t)$  it results the function of the normally unitary effort represented at the upper of the rail fiber, or fiber lower flange, figure 11 and figure 12.

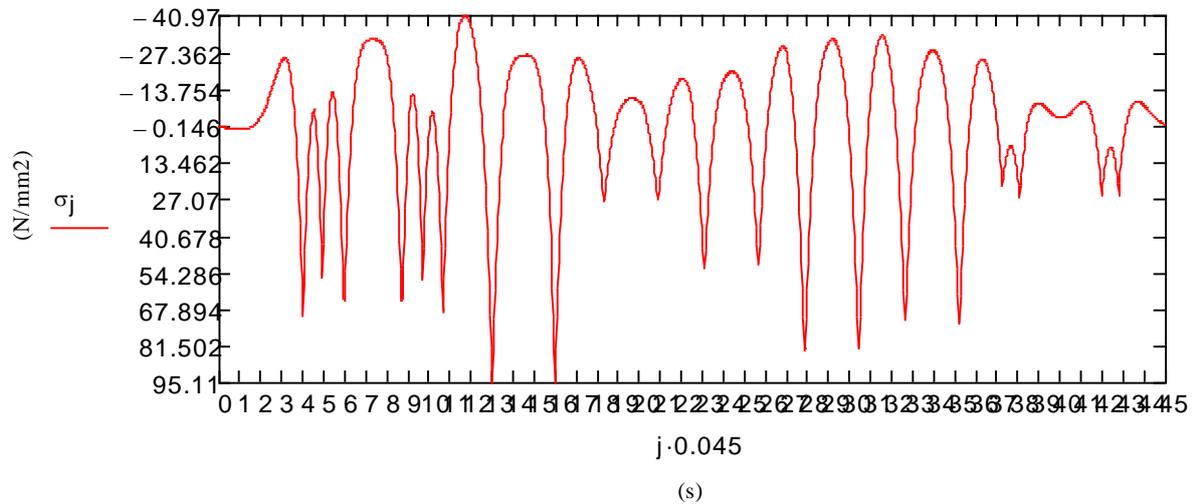


Figure 11. Normal unitary effort at the upper fiber rail versus time, for general freight train

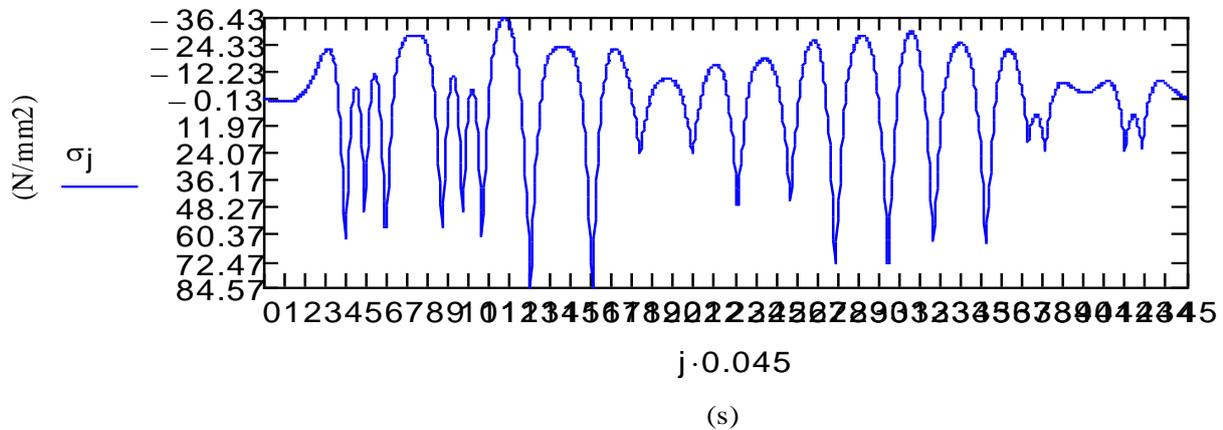


Figure 12. Normal unitary effort to lower fiber rail versus time, for general freight train

## 5. Conclusions

For an accurate knowledge of railway track request it is necessary the study of variation according to the time, while browsing the convoys area (trains). Determination of calculating characteristics according to the random requests are necessary for real knowledge of how to apply for the track to determine the projected life. In determining the life of the railway tracks it should be considered and accepted the service wear.

## REFERENCES

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